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Fingero Imbibition Phenomenon Through Porous Media Under Magnetic Field Effects

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A B S T R A C T

The present paper numerically discusses the fingero–imbibition phenomena through homogeneous porous medium with the involvement of a layer of magnetic fluid in the injected phase. The basic assumptions underlying the present investigation are that the two fluids are immiscible and the injected fluid is less viscous as well as preferentially wetting with respect to porous materials and with capillary pressure. A numerical solution of the governing non-linear partial differential equation has been obtained by two parameter Ritz approximation method [6].The graphs of space and time versus saturation of injected fluid of numerical results have been obtained which indicates the stabilization of fingers.

AMS Subject classification: 76A05, 76M55, 54H15

Nomenclature :

V_i =velocity of the injected liquid

V_n= velocity of the native liquid

K = permeability of medium

K_i= relative permeability of the injected liquid

K_n = relative permeability of the native liquid

P_i = pressure of the injected liquid

P_n = pressure of the native liquid

S_i = saturation of the injected liquid (water),

S_n = saturation of the native liquid (oil)

t= time

P_c=capillary pressure

P = porosity of the porous medium

δ_i = viscosity of the injected liquid,

δ_n = viscosity of the native liquid

1. Introduction

It is well known that if a porous medium filled with some fluid is brought into contact with another fluid which preferentially wets the medium, then there is a spontaneous flow of the wetting fluid into the medium and a counter flow of the resident fluid from the medium. This phenomenon is called an imbibition [7]. Again, when a fluid contained in a porous medium is displaced by another of lesser viscosity, instead of a regular displacement of the whole front, protuberances (fingers) may occur that shoot through the porous medium at relatively great speeds. This phenomenon is called fingering [7].

Under certain circumstances the phenomena of fingering and imbibition may occur simultaneously in a displacement process involving two immiscible fluids and are referred to as finger-imbibition phenomenon. The phenomena of fingering and imbibition whether occurring singly or simultaneously in displacement process, have gained much importance in the problems of petroleum technology and hydrogeology and many authors have investigated them from different aspects; for example, Grahm and Richrdson [1], Scheidegger [8-10], Verma [12-17], Mehta [4], Mishra [3], Rijik [5].

The present paper numerically discusses the fingero–imbibition phenomena through homogeneous porous medium with the involvement of a layer of magnetic fluid in the injected phase. The basic assumptions underlying the present investigation are that the two fluids are immiscible and the injected fluid is less viscous as well as preferentially wetting with respect to porous materials and with capillary pressure. A numerical solution of the governing non-linear partial differential equation has been obtained by two parameter Ritz approximation method [6].The graphs of space and time versus saturation of injected fluid of numerical results have been obtained which indicates the stabilization of fingers.

2.Mathematical Formulation

It is considered here, that a finite cylindrical piece of porous matrix saturated with native liquid (N) is completely surrounded by an impermeable surface (x = 0) and this end is exposed to an adjacent formation of injected liquid (I) which involves a thin layer of magnetic fluid. It is assumed that the later fluid is preferentially wetting and less viscous. This arrangement gives rise to a displacement process in which the injection of the fluid (I) is initiated by imbibition and the consequent displacement of native liquid (N) produces protuberances (fingers). This arrangement describes a one – dimensional phenomenon of fingero – imbibition.

Assuming that the flow of two immiscible phases is governed by Darcy’s law, we may write the seepage velocities of the injected and native liquids as,

$$V_i = -\frac{K_i}{\delta_i} K \left(\frac{\partial P_i}{\partial x} + \alpha H \frac{\partial H}{\partial x} \right) \quad (1)$$

$$V_n = -\frac{K_n}{\delta_n} K \left(\frac{\partial P_n}{\partial x} \right) \quad (2)$$

Where

$$\alpha = \frac{\mu}{4\pi} = \mu_0\lambda + \frac{16\pi\ \mu_0\ \lambda^2 \mathbf{r}^3}{g\ (1+2)^3}$$

$$P\left(\frac{\partial S_i}{\partial t}\right) + \left(\frac{\partial V_i}{\partial \mathbf{x}}\right) = 0 \tag{3}$$

$$P\left(\frac{\partial S_n}{\partial t}\right) + \left(\frac{\partial V_n}{\partial \mathbf{x}}\right) = 0 \tag{4}$$

$$V_i + V_n = 0 \tag{5}$$

$$P_c = P_n - P_i \tag{6}$$

It may be mentioned that the term $\alpha\ H\left(\frac{\partial H}{\partial \mathbf{x}}\right)$

On the right hand side of equation(1) presents the additional pressure exerted due to the pressure of a layer of magnetic fluid in the displacing phase (i) .combining equation (1) to (5) and then using the relation .

$$\frac{\frac{K_i}{\delta_i}\ \frac{K_n}{\delta_n}}{\left(\frac{K_i}{\delta_i} + \frac{K_n}{\delta_n}\right)} \approx \frac{K_n}{\delta_n} \tag{7}$$

By Scheidegger [11] ,we have equation obtained ,

$$P\left(\frac{\partial S_i}{\partial t}\right) + \frac{\partial}{\partial \mathbf{x}}\left[\frac{K_n \cdot K}{\delta_n}\left(\frac{\partial P_c}{\partial \mathbf{x}} - \alpha\ H\ \frac{\partial H}{\partial \mathbf{x}}\right)\right] = 0 \tag{8}$$

For definiteness, the following relationships have been assumed

$$K_i = S_i$$

$$K_n = S_n = 1 - S_i \tag{9}$$

for the statistical treatment of fingers due to Scheidegger and Johnson [11].

$Pc = -\beta\ Si$ due to Muskat [4]. (10)

By using above assumption, equation (8) becomes,

$$P\left(\frac{\partial S_i}{\partial t}\right) + \frac{\partial}{\partial x}\left[K\frac{(1-S_i)}{\delta_n}\left(-\beta\frac{\partial S_i}{\partial x} - \alpha H\frac{\partial H}{\partial x}\right)\right] = 0 \qquad (11)$$

Considering

$1 - Si(x, t) = S(x, t)$

And the magnetic field H in the X direction [14]

$$P\left(\frac{\partial S}{\partial t}\right) - \frac{K\beta}{\delta_n}\frac{\partial}{\partial x}\left(S\frac{\partial S}{\partial x}\right) - \frac{\alpha K\lambda^2 n}{\delta_n}\frac{\partial}{\partial x}\left(\frac{S}{x^{2n+1}}\right) = 0 \qquad (12)$$

Where λ is a constant parameter, Choose due to Mehta [2]

$n = -1/2$

Equation (12) obtained as

$$\left(\frac{\partial S}{\partial t}\right) - \alpha_1 S\frac{\partial^2 S}{\partial x^2} - \alpha_1\left(\frac{\partial S}{\partial x}\right)^2 + \alpha_2\frac{\partial S}{\partial x} = 0 \qquad (13)$$

Where

$$\alpha_1 = \frac{K\beta}{\delta_n P}; \qquad \alpha_2 = \frac{\alpha K\lambda^2}{2\delta_n P}$$

This is the non linear partial differential equation governing the fingero imbibition phenomenon in a double phase flow through homogeneous porous media.A set of suitable initial and boundary condition associated with problem are ,

$S(0, t) = S_0$ at $x = 0;\ t > 0$

$S(1, t) = S_1$ at $x = 1;\ t > 0$

$S(x, 0) = \theta\varepsilon << 1$ at $x > 0;\ t = 0$

$$\left.\frac{\partial S}{\partial x}\right|_{x=0} = 0, \qquad \left.\frac{\partial S}{\partial x}\right|_{x=1} = 0; \qquad t \ge 0 \qquad (14)$$

3. Solution by Ritz method

The weak form of equation (13) is

$$0 = \int_0^1 V \frac{\partial S}{\partial t}\, dx + \alpha_1 \int_0^1 S \frac{\partial V}{\partial x} \frac{\partial S}{\partial x}\, dx - \alpha_2 \int_0^1 V \frac{\partial S}{\partial x}\, dx - \alpha_1 \left(S\, V \frac{\partial S}{\partial x} \right) \Bigg|_{x=0}^{x=1} \qquad (15)$$

Putting $V = \phi_i$; $i = 1,2$ which satisfies the boundary conditions (14), it is an arbitrary continuous function. The two parameter Ritz approximation function is

$$S(x, t) = C_1(t)\, \phi_1(x) + C_2(t)\, \phi_2(x) \qquad (16)$$

Where $\phi_1(x) = S_0 + (S_1 - S_0)\, x$ (17)

$$\phi_2(x) = S_0 + (S_1 - S_0)\, x^2 \qquad (18)$$

Ritz equations are

$$C_1' \left[\int_0^1 \phi_1^2\, dx \right] + C_2' \left[\int_0^1 \phi_1 \phi_2\, dx \right]$$

$$+ \alpha_1 \left[C_2^2 \int_0^1 \phi_1' \phi_2' \phi_2\, dx + C_1^2 \int_0^1 \phi_1'^2 \phi_1\, dx + C_1 C_2 \int_0^1 (\phi_2' \phi_1 + \phi_1' \phi_2)\, \phi_1'\, dx \right]$$

$$- \alpha_2 \left[C_1 \int_0^1 \phi_1 \phi_1'\, dx + C_2 \int_0^1 \phi_2' \phi_1\, dx \right]$$

$$+ \alpha_1 \left[S\,\phi_1 \frac{\partial S}{\partial x}\Big|_{x=1} - S\,\phi_1 \frac{\partial S}{\partial x}\Big|_{x=0} \right] = 0 \qquad (19)$$

and

$$C_1' \left[\int_0^1 \phi_1 \phi_2 \, dx \right] + C_2' \left[\int_0^1 \phi_2^2 \, dx \right]$$

$$+ \alpha_1 \left[C_2^2 \int_0^1 \phi_2 \phi_2^{2\prime} \, dx + C_1^2 \int_0^1 \phi_1' \phi_1 \phi_2 \, dx + C_1 C_2 \int_0^1 (\phi_2' \phi_1 + \phi_1' \phi_2)\, \phi_2' \, dx \right]$$

$$- \alpha_2 \left[C_1 \int_0^1 \phi_2 \phi_1' \, dx + C_2 \int_0^1 \phi_2' \phi_2 \, dx \right]$$

$$+ \alpha_1 \left[S\,\phi_2 \frac{\partial S}{\partial x}\Big|_{x=1} - S\,\phi_2 \frac{\partial S}{\partial x}\Big|_{x=0} \right] = 0 \qquad (20)$$

for i = 1 and i = 2 respectively.

Where $C_i' = \dfrac{\partial C_i}{\partial t}$ and $\phi_i' = \dfrac{\partial \phi_i}{\partial x}$

The equations (19) and (20) are rewritten into the from of

$$C_1' \left(\frac{S_1^2 + S_0 S_1 + S_0^2}{3} \right) + C_2' \left(\frac{5S_0^2 + 4S_0 S_1 + 3S_1^2}{12} \right) + \alpha_1 \frac{(C_1^2 + C_2^2)(S_1 - S_0)^2 (S_1 + S_0)}{2}$$

$$+ C_1 C_2 (S_1 - S_0)^2 (S_1 + S_0) - \alpha_2 C_1 \frac{(S_1^2 - S_0^2)}{2} - \alpha_2 C_2 \frac{(S_1 - S_0)(2S_1 + S_0)}{3} = 0 \qquad (21)$$

and

$$C_1' \left(\frac{5S_0^2 + 4S_0 S_1 + 3S_1^2}{12} \right) + C2' \left(\frac{3S_1^2 + 4S_0 S_1 + 8S_0^2}{15} \right) + \alpha_1 \frac{C_1^2 (S_1 - S_0)^2 (2S_1 + S_0}{3}$$

$$+ 4\alpha_1 C_2^2 \frac{(S_1 - S_0)^2 (S_1 + S_0)}{3} + C_1 C_2 \frac{(S_1 - S_0)^2 (6S_1 + 4S_0)}{6}$$

$$- \alpha_2 C_1 \frac{(S_1 - S_0)(S_1 + 2S_0)}{3} - \alpha_2 C_2 \frac{(S_0^2 - S_1^2)}{2} = 0 \quad (22)$$

The residual of the approximation in the initial condition is

$$Y = S(x, 0) - \theta\varepsilon \quad (23)$$

Using Galerkin method, we get

$$\int_0^1 [S(\mathbf{x}, 0) - \theta\varepsilon] \phi i = 0 \;; \quad \text{for } i = 1, 2 \quad (24)$$

Obtaining the equations

$$C_1(0) \left(\frac{S_1^2 + S_0 S_1 + S_0^2}{3} \right) + C_2(0) \left(\frac{5S_0^2 + 4S_0 S_1 + 3S_1^2}{12} \right) - \theta\varepsilon \frac{(S_1 + S_0)}{2} = 0 \quad (25)$$

and

$$C_1(0) \left(\frac{5S_0^2 + 4S_0 S_1 + 3S_1^2}{12} \right) + C_2(0) \left(\frac{3S_1^2 + 4S_0 S_1 + 8S_0^2}{15} \right) - \theta\varepsilon \frac{(S_1 + 2S_0)}{3} = 0 \quad (26)$$

Taking Laplace transform to the equations (21) and (22) and solving them with the help of Ritz co-efficient,

$$C_1(0) \cong 0.0043913 \;; \qquad C_2(0) \cong -0.0035032 \quad (27)$$

We get,

$$C_1(t) = e^{(-40.826738 \cdot t)} \left\{ \begin{array}{l} (0.1069225) \cosh(42.16343 \cdot t) \\ -(0.0077116) \sinh(42.16343 \cdot t) \end{array} \right\} \quad (28)$$

And

$$C_2(t) = e^{(-40.826738 \cdot t)} \left\{ \begin{array}{l} (0.1225254)\sinh(42.16343 \cdot t) \\ -(0.134899)\cosh(42.16343 \cdot t) \end{array} \right\} \tag{29}$$

The solution of the given problem equation (13) is of the form,

$$S(\mathbf{x}, t) = e^{(-40.826738 \cdot t)} \left\{ \begin{array}{l} \cosh(42.16343 \cdot t) \cdot \\ (0.0093433 + 0.085538 \cdot \mathbf{x} - 0.01079 \cdot \mathbf{x}^2) \\ + \sinh(42.16343 \cdot t) \cdot \end{array} \right\} \tag{30}$$

The following values of various parameters have been considered in the present analysis:

$S_0 = 0.1$, $S_1 = 0.9$, $\theta\varepsilon = 0.001$, $\alpha_1 = 0.125$, $\alpha_2 = 0.137$

Table 1 : Graphical and numerical representation :

t	x = 0.1	x = 0.3	x = 0.5	x = 0.7	x = 0.9
0	0.01779	0.03403	0.04941	0.06393	0.07759
0.1	0.02232	0.04273	0.06257	0.08184	0.10054
0.2	0.02551	0.04884	0.07152	0.09354	0.11492
0.3	0.02916	0.05583	0.08175	0.10692	0.13135
0.4	0.03333	0.06381	0.09344	0.12221	0.15014
0.5	0.03810	0.07294	0.10680	0.13969	0.17161
0.6	0.04355	0.08337	0.12208	0.15967	0.19615
0.7	0.04978	0.09529	0.13953	0.18250	0.22420
0.8	0.05689	0.10892	0.15949	0.20861	0.25627
0.9	0.06503	0.12450	0.18230	0.23844	0.29292
1	0.07433	0.14230	0.20837	0.27254	0.33481

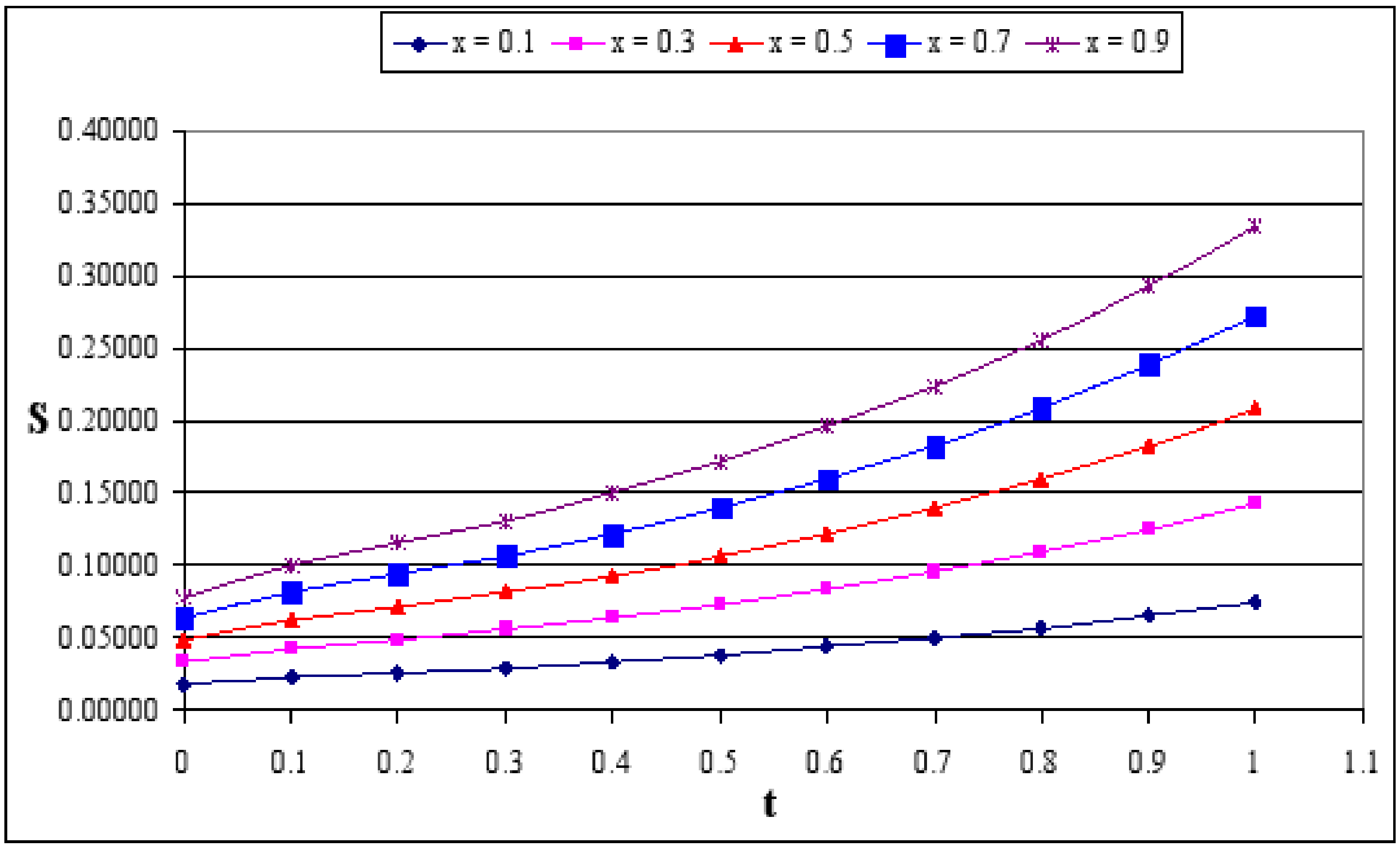

Figure-1

From the Figure-1 it is clear that S is increasing parabolically as t increases which is quite relevant to the physical fact for imbibition phenomenon. In this phenomenon, first imbibition occurs then fingering. Therefore due to fingering the saturation will increase as S and t increases.

Table -2

x	t = 0.1	t = 0.3	t = 0.5	t = 0.7	t = 0.9
0	0.01190	0.01555	0.02031	0.02654	0.03467
0.1	0.02232	0.02916	0.03810	0.04978	0.06503
0.2	0.03260	0.04259	0.05564	0.07269	0.09497
0.3	0.04273	0.05583	0.07294	0.09529	0.12450
0.4	0.05272	0.06888	0.08999	0.11757	0.15361
0.5	0.06257	0.08175	0.10680	0.13953	0.18230
0.6	0.07227	0.09443	0.12337	0.16118	0.21058
0.7	0.08184	0.10692	0.13969	0.18250	0.23844
0.8	0.09126	0.11923	0.15577	0.20351	0.26589
0.9	0.10054	0.13135	0.17161	0.22420	0.29292
1	0.10967	0.14329	0.18720	0.24457	0.31953

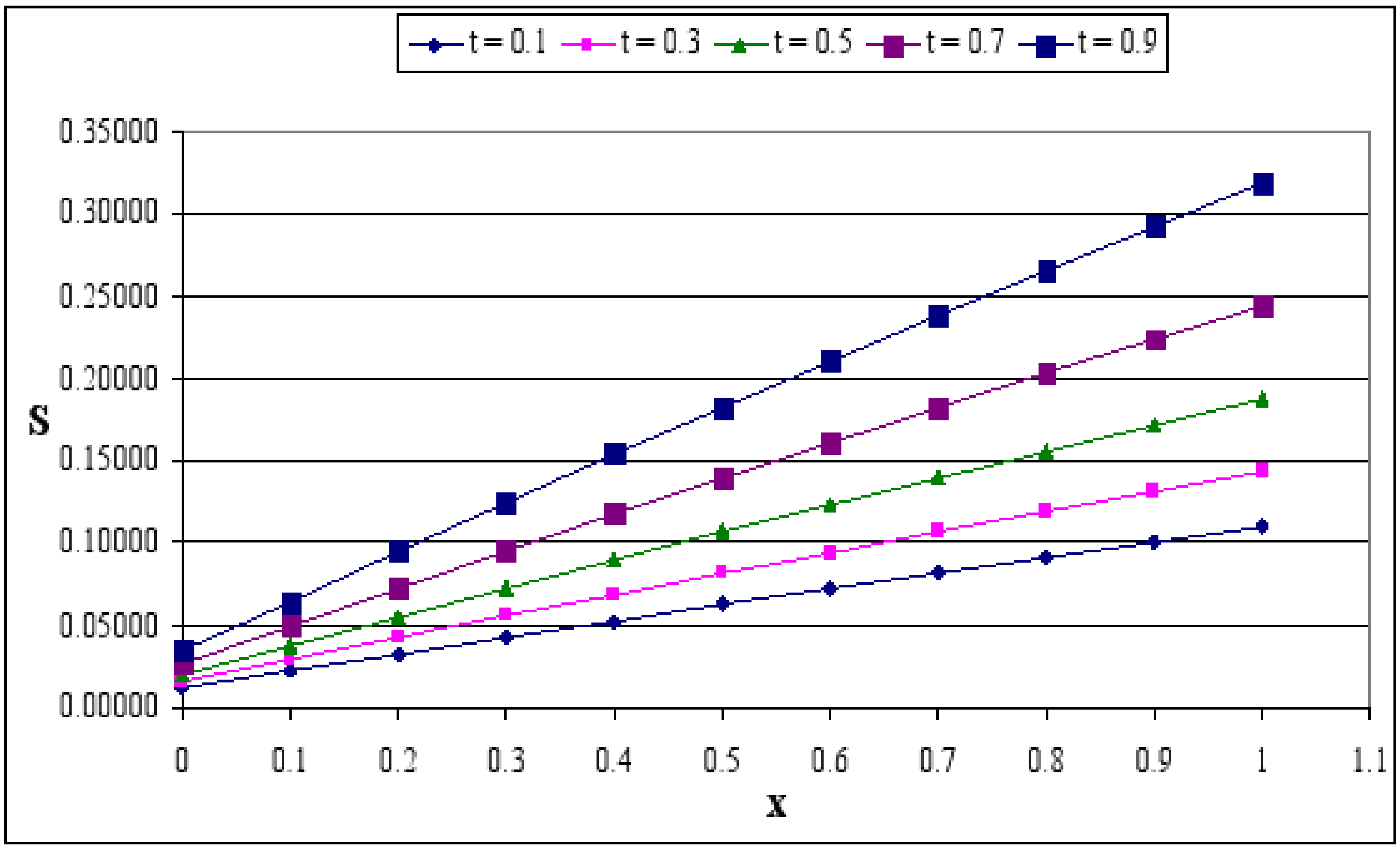

Figure-2

The graph Figure-2 x versus S linearly increases as x increases. The graph initially start from $S = S_0$ when $x = x_0$. At different times if t increases, saturation increases linearly.

Some specific values of parameter have been considered as per standard literature available. From graph we can study the nature of the phenomenon.

4. Conclusion

Here, we have discussed a numerical solution of a mathematical model of fingero imbibition phenomenon in a homogeneous porous medium involving magnetic field with using a two parameter Ritz approximation technique. Equation (30) is a resultant solution for saturation in terms of exponential and hyperbolic functions.

For the sake of convenience in the mathematical analysis, some standard relations of saturation and capillary pressure are assumed, which are consistent with physical problem. The solution of injected liquid is obtained under assumption that average cross sectional area is occupied by the fingers. The numerical as well as graphical illustrations are given by Table-1, Figure-1, Table-2 and Figure-2.

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