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THE EFFECT OF CHEMICAL REACTION ON AN MHD BOUNDARY LAYER FLOW PAST A STREATCHING SHEET

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A B S T R A C T

Steady two dimensional Magnetohydrodynamic (MHD) laminar incompressible boundary layer flows past a stretching sheet with diffusion and chemical reaction. The system of non-linear partial differential equations transformed in to an ordinary non-linear differential equation by group symmetry method. The fluid considered in the study is the Newtonian fluid. Representative results are in several graphs to steady an effect of Schmidt numberS_c, reaction parameter δ, and magnetic field strength M on the concentration.

Introduction

Investigations of laminar boundary flow of an electrically conducting fluid over a moving continuous stretching surface are important in many manufacturing processes, such as materials manufactured by polymer extrusion, continuous stretching of plastic films, artificial fibers, hot rolling, wire drawing, glass fiber, metal extrusion and metal spinning, cooling of metallic sheets, or electronic chips. The first and foremost work regarding the boundary-layer behavior in moving surfaces in quiescent fluid was considered by Sakiadis [1]. In many process industries, the cooling of threads or sheets of some polymer materials is of importance in the production line. The rate of cooling can be controlled effectively to achieve final products of desired characteristics by drawing threads, etc. in the presence of an electrically conducting fluid subject to a magnetic field.

MHD viscous flow over a stretching sheet in the presence of slip velocity was studied by many authors, such as Martin and Boyd [2], Fang and Lee [3], Ariel et al. [4], Andersson[5],Wang [6, 7], Mukhopadhyay and Anderson [8], Fang[9], and Hayat et al. [10]. Fang et al. [11] studied MHD viscous flow over a permeable shrinking sheet. They observed that the velocity at the wall increased with slip parameter. Mahmoud and Waheed [12] included the effects of slip and heat generation/absorption on MHD mixed convection flow of a micro-polar fluid over a heated stretching surface.

Darji and Timol [13] studied the Deductive group theoretic analysis for MHD flow a Sisko fluid in a porous medium. Anderson et al. [14] studied the diffusion of a chemically reactive species from a stretching sheet. The study of constant MHD hall effect for free convective flow and mass transfer over a stretching sheet with chemical reaction was done by Afify [15].The similarity solutions of mixed convection with diffusion and chemical reaction over a horizontal moving plate were obtained by Fan et al. [16].

Thermal radiation effects on an electrically conducting fluid arise in many practical applications such as electrical power generation; solar power technology, nuclear reactors, and nuclear waste disposal (see Mahmoud [17] and Chamkha [18]. Heat and mass transfer problems with a chemical reaction have received a considerable amount of attention due to their importance in many chemical engineering processes, for example, food processing, manufacturing of ceramics, and polymer production.

In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications viz., polymer production, manufacturing of ceramics or glassware and food processing. Chambre and Young [19] have presented a first order chemical reaction in the neighborhood of a horizontal plate. Radiation and chemical reaction effects on an unsteady MHD convection flow past a vertical moving porous plate embedded in a porous medium with viscous dissipation was analyzed by Babu et al [20].

Darji and Timol [21] investigated electrically conducting Magnetohydrodynamic (MHD) laminar incompressible boundary layer for the Newtonian fluid past a stretching sheet with diffusion and chemical reaction. Mukhopadhyay et al. [22] studied the MHD boundary layer flow and heat transfer over a stretching sheet with variable viscosity using the scaling group of transformations.

So motivated by the previous works, the aim of the present work is to analyze the steady two dimensional Magnetohydrodynamic (MHD) laminar incompressible boundary layer flows past a stretching sheet with diffusion and chemical reaction. The system of non-linear partial differential equations transformed in to an ordinary non-linear differential equation by linear group of transformation. The fluid considered in the study is the Newtonian fluid. Representative results are in several graphs to steady an effect of Schmidt number S_c, reaction parameter δ , and magnetic field strength M on the concentration.

Mathematical Analysis

Consider the physical situation for a steady state, laminar boundary layer flow of an electrically conducting incompressible viscous fluid in the presence of a transverse magnetic field B_0 due to a stretching horizontal sheet. The flow is generated by the action of two equal and opposite forces along the *X*-axis and the sheet is stretched with a velocity that is proportional to the distance from the slit. The concentration at the stretching sheet is $C_w(x) = kx$ where *k* is constant. The stretching sheet assumed velocity of the form $U_w(x) = bx$ where *b* is the stretching constant and *x* is the distance from the slit. It is also assumed that the magnetic Reynolds number Re_m is very small. Hence the governing equations of the problem are:

Momentum equation

$$\mathbf{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u} \tag{1}$$

Concentration equation

$$\mathbf{u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - kC} \tag{2}$$

With boundary conditions

$$\mathbf{y = 0}, \qquad \mathbf{u = U_w(= bx)\,, v = 0}$$

$$\mathbf{C = C_w(x)\ \ (= kx)}$$

$$\mathbf{y = \infty, \qquad u = 0\ , \qquad C(x,y) = 0} \tag{3}$$

We introduce the following dimensionless quantity,

$$\mathbf{x^* = \frac{x}{L}\ , \qquad y^* = \frac{y}{L}\sqrt{R_e}\,, \qquad u^* = \frac{u}{U_0}\ , \qquad v^* = \frac{v}{U_0}\sqrt{R_e}\,, \qquad C^* = \frac{C}{C_0}} \tag{4}$$

Where $U_0 = bL$-reference moving speed, $R_e = \frac{U_0 L}{v}$ -Reynolds number, $C_0 = cL$ -reference concentration. Dropping the asterisks (for simplicity) the boundary layer equations become

Momentum equation

$$\mathbf{u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \frac{\partial^2 u^*}{\partial {y^*}^2} - Mu^*} \tag{5}$$

Concentration equation

$$\mathbf{u^*\frac{\partial C^*}{\partial x^*} + v^*\frac{\partial C^*}{\partial y^*} = \frac{1}{Sc}\frac{\partial^2 C^*}{\partial {y^*}^2} - \delta C^*} \tag{6}$$

Where $Sc = \frac{v}{D}$- Schmidt's number, $\delta = \frac{kL}{U_0}$ -Reaction rate parameter, $M = \frac{\sigma B_0^2}{\rho}$- magnetic field strength parameter

Also the boundary conditions become

$$\mathbf{y = 0\,, \qquad u^*(x,0) = x, \qquad v^*(x,0) = 0, \qquad C(x,0) = x}$$

$$\mathbf{y \to \infty\,, \qquad u^*(x,y) = 0\,, \qquad C^*(x,y) = 0} \tag{7}$$

Introducing stream function ψ such that,

$$\mathbf{u^* = \frac{\partial \psi^*}{\partial y^*} \qquad , v^* = -\frac{\partial \psi^*}{\partial x^*}} \tag{8}$$

Equations (4.5-4.6) become

$$\mathbf{\frac{\partial \psi^*}{\partial y^*}\frac{\partial^2 \psi^*}{\partial x^* \partial y^*} - \frac{\partial \psi^*}{\partial x^*}\frac{\partial^2 \psi^*}{\partial {y^*}^2} = \frac{\partial^3 \psi}{\partial y^3} - M\frac{\partial \psi^*}{\partial y^*}} \tag{9}$$

$$\mathbf{\frac{\partial \psi^*}{\partial y^*}\frac{\partial c^*}{\partial x^*} - \frac{\partial \psi^*}{\partial x^*}\frac{\partial c^*}{\partial y^*} = \frac{1}{Sc}\frac{\partial^2 c^*}{\partial {y^*}^2} - \delta C} \tag{10}$$

With boundary conditions

$$\mathbf{y = 0}, \qquad \frac{\partial \psi^*}{\partial y^*} = \mathbf{x}, \qquad \frac{\partial \psi^*}{\partial x^*} = \mathbf{0}, \qquad \mathbf{C^* = x}$$

$$\mathbf{y = \infty}, \qquad \frac{\partial \psi^*}{\partial y^*} = \mathbf{0}, \qquad \mathbf{C^*(x, y) = 0} \tag{11}$$

Method of solution

By using linear group transformation,

$$\bar{x}^* = P^{A_1} x^*, \qquad \bar{y}^* = P^{A_2} y^*, \qquad \bar{\psi}^* = P^{A_3} \psi^*, \quad \bar{C}^* = P^{A_4} C^* \tag{12}$$

Where A_1, A_2, A_3, A_4 and P are constants

For the dependent and independent variables. From equation (12) one obtains

$$\left(\frac{\bar{x}^*}{x^*}\right)^{\frac{1}{A_1}} = \left(\frac{\bar{y}^*}{y^*}\right)^{\frac{1}{A_2}} = \left(\frac{\bar{\psi}^*}{\psi^*}\right)^{\frac{1}{A_3}} = \left(\frac{\bar{C}^*}{C^*}\right)^{\frac{1}{A_4}} = P \tag{13}$$

Introducing the linear transformation, given by equation (13), in to the equations (9-10) results in

$$P^{A_1+2A_2-2A_3} \frac{\partial \bar{\psi}^*}{\partial \bar{y}^*} \frac{\partial^2 \bar{\psi}^*}{\partial \bar{x}^* \partial \bar{y}^*} - P^{A_1+2A_2-2A_3} \frac{\partial \bar{\psi}^*}{\partial \bar{x}^*} \frac{\partial^2 \bar{\psi}^*}{\partial {\bar{y}^*}^2}$$

$$= P^{3A_2-A_3} \frac{\partial^3 \bar{\psi}}{\partial {\bar{y}^*}^3} - M P^{A_2-A_3} \frac{\partial \bar{\psi}^*}{\partial \bar{y}^*} \tag{14}$$

$$P^{A_1+A_2-A_3-A_4} \frac{\partial \bar{\psi}^*}{\partial \bar{y}^*} \frac{\partial \bar{C}^*}{\partial \bar{x}^*} - P^{A_1+A_2-A_3-A_4} \frac{\partial \bar{\psi}^*}{\partial \bar{x}^*} \frac{\partial \bar{C}^*}{\partial \bar{y}^*}$$

$$= \frac{1}{Sc} P^{2A_2-A_4} \frac{\partial^2 \bar{C}^*}{\partial {\bar{y}^*}^2} - \delta P^{-A_4} \bar{C} \tag{15}$$

The differential equation are completely invariant to the proposed linear transformation, the following coupled algebraic equations are obtained

$$A_1 + 2A_2 - 2A_3 = A_2 - A_3 \tag{16}$$

$$3A_2 - A_3 \qquad = A_2 - A_3 \tag{17}$$

$$\mathbf{A_1 + A_2 - A_3 - A_4 = 2A_2 - A_4} \tag{18}$$

$$\mathbf{A_4 = 0} \tag{19}$$

By solving above equations (16-19), we get

$$\mathbf{A_2 = 0}, \qquad \mathbf{A_1 = A_3}, \qquad \mathbf{A_4 = 0} \tag{20}$$

Divide throughout by $\mathbf{A_1}$,We get

$$\frac{\mathbf{A_2}}{\mathbf{A_1}} = \mathbf{0}, \qquad \frac{\mathbf{A_3}}{\mathbf{A_1}} = \mathbf{0}, \qquad \frac{\mathbf{A_4}}{\mathbf{A_1}} = \mathbf{0} \tag{21}$$

Introducing equation (21) in to equation (13) result in

$$\boldsymbol{\eta} = \mathbf{y^*}, \qquad \boldsymbol{\psi}^* = \mathbf{f(\boldsymbol{\eta})x^*}, \qquad \mathbf{G(\boldsymbol{\eta}) = C} \tag{22}$$

The reduction to an ordinary differential equation

Transforming the individual terms into equations (9-11) result in

$$\frac{\partial \psi^*}{\partial y^*} = x.f'(\eta), \qquad \frac{\partial \psi^*}{\partial x^*} = f(\eta)$$

$$\frac{\partial^2 \psi^*}{\partial y^{*2}} = x.f''(\eta), \qquad \frac{\partial^2 \psi^*}{\partial x^* \partial y^*} = f'(\eta)$$

$$\frac{\partial^3 \psi^*}{\partial y^{*3}} = x.f'''(\eta), \qquad \frac{\partial C^*}{\partial x^*} = 0$$

$$\frac{\partial C^*}{\partial y^*} = G'(\eta), \qquad \frac{\partial^2 C^*}{\partial y^{*2}} = G''(\eta) \tag{23}$$

These expressions inserting into equations (9-11), we get

$$x.f'(\eta).f'(\eta) - f(\eta).x.f''(\eta) = x.f'''(\eta) - M.x.f'(\eta)$$

$$f'''(\eta) + f(\eta).f''(\eta) - f'^2(\eta) - M.f'(\eta) = 0 \tag{24}$$

From equation (10), we get

$$x.f'(\eta).0 - f(\eta)G'(\eta) = \frac{1}{Sc}.G''(\eta) - \delta G(\eta)$$

$$\frac{1}{Sc}.G''(\eta) + f(\eta)G'(\eta) - \delta G(\eta) = 0 \tag{25}$$

Together with boundary conditions,

$$\eta = 0, \quad f(0) = 0, \quad f'(0) = 1, \quad G(0) = 1$$

$$\eta \to \infty, \quad f'(\infty) = 0, \quad G(\infty) = 0 \tag{26}$$

Physical quantities of interest:

The momentum boundary layer equation (24) subjected to the boundary conditions (26) has an exact solution (see Pavlov [23]) of the form

$$f(\eta) = \frac{1-e^{-m\eta}}{m} \qquad \textbf{Where} \qquad m = \sqrt{1+M}$$

(27)

The similarity function related to horizontal velocity is given by

$$f'(\eta) = e^{-m\eta}$$

The skin-friction coefficient C_{fx} is given by

$$f''(0) = -\sqrt{1+M}$$

$$\frac{1}{2}\sqrt{Re_x}C_{fx} = -\sqrt{1+M} \tag{28}$$

Where Re_x is the local Reynolds number. Equation (25) subject to the boundary conditions (25) becomes

$$\frac{1}{Sc}.G''(\eta) + \left(\frac{1-e^{-m\eta}}{m}\right)G'(\eta) - \delta G(\eta) = 0 \tag{29}$$

Result and Discussions

To solve the boundary value problems by the collocation method Eq. (29) is transformed into a system of two first order differential equations as

$$f_0'(\eta) = f_1 \;\; and \;\; f_1'(\eta) = f_2 = S_c\left(\delta f_0 - \left(\frac{1-e^{-m\eta}}{m}\right)f_1\right) \tag{30}$$

The boundary conditions (26) as

$$\mathbf{f_0(0) = 0}\ , \qquad \mathbf{f_0(\infty) = 1} \quad \boldsymbol{Where}\ \mathbf{f_0 = g(\boldsymbol{\eta})} \tag{31}$$

We use the collocation method (see [24]) in finding the numerical solutions of the resulting boundary value problems. The numerical procedure is treated using computational appropriate algorithm in MatLab.

Equations (29) is transformed into a system of non-linear algebraic equations (finite difference equations) using a central difference scheme with uniform mesh points. The boundary conditions in (29) form a part of the system of finite difference equations. The transformed system of non-linear algebraic equations is then linearized by Newton's method. This system of linear algebraic equations is then solved by the Gauss elimination method. Shooting method (see [25]) is used to obtain the initial guess solution.

The results are presented in several graphs under an effect of Schmidt number *Sc*, reaction parameter δ and magnetic field strength *M* on the concentration.

In is observed that analytical solution for the momentum boundary layer equation shows that the horizontal velocity component is exponentially decreased where as the local skin friction coefficient is increased as the magnetic field strength increased.

From the fig.1, the effect of Schmidt number by controlling the magnetic field strength and concentration parameter. It shows that the concentration decreases as the Schmidt number increase that warrants the fact of chemical reaction phenomenon.

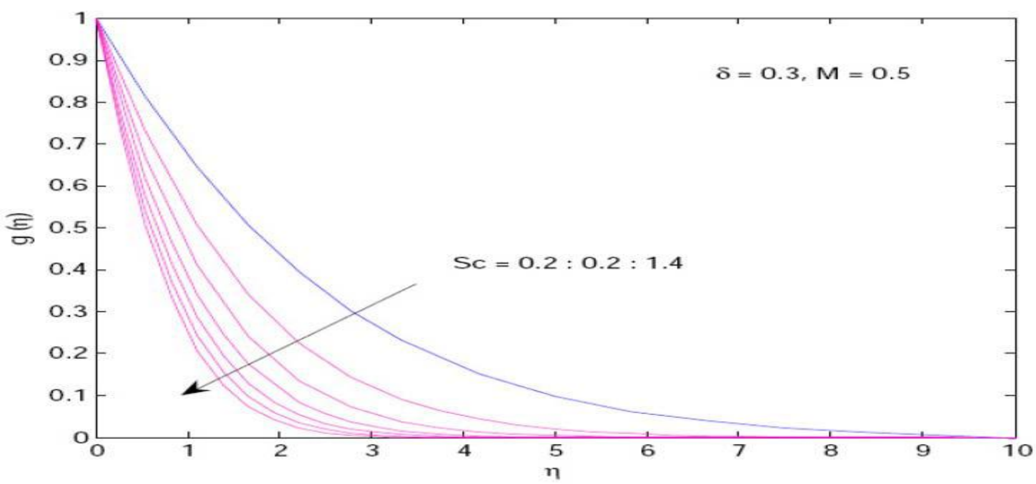

Fig. 1: Effect of Schmidt number S_c on dimensionless concentration δ

Fig. 2 projects the intensity of magnetic field on the concentration. From this profile it is evident that as the intensity of the magnetic field parameter M increases the chemical concentration δ of fluid is sharply increase. It shows the importance of magnetic field during chemical reaction.

Fig. 3 depicts the influence of reaction parameter. This graph suggests that in presence of magnetic field M and controlling the Schmidt numberS_c, an increase in reaction parameter δ will decrease the chemical concentration. Fig. 1 and Fig. 2 display the co-related effects of Schmidt number and reaction parameter in presence of magnetic field M

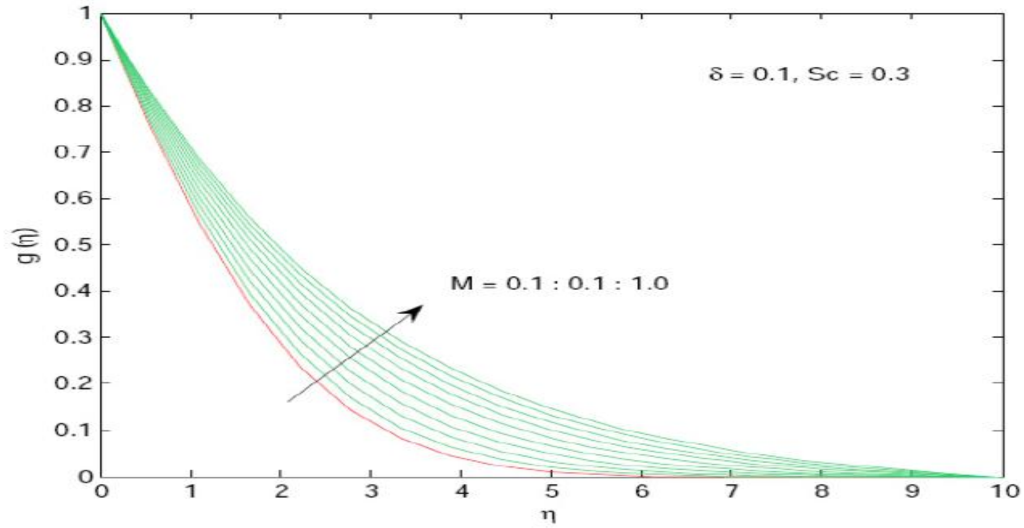

Fig. 2: Intensity of magnetic strength M on concentration

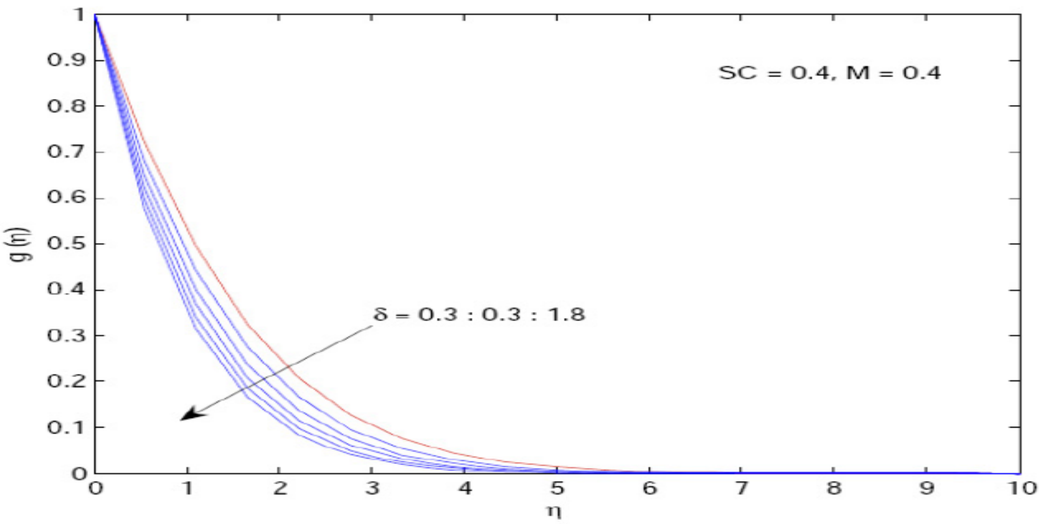

Fig 4: Influence of reaction parameter on concentration

Conclusion

We have used Linear-group of transformation method to obtain the similarity reductions of the MHD boundary layer flow equations past a stretching sheet with diffusion and chemical reaction in an electrically conducting Newtonian fluid .By determining the transformation group under which the given system of non-linear partial differential equations and its boundary conditions are invariant, we obtained the invariants and the symmetries of these equations. In turn, we used these invariants and symmetries to determine the similarity variables that reduced the number of independent variables. The resulting system of ordinary non-linear differential equations was solved numerically using collocation method and the results were plotted. It is observed that the concentration of the fluid is decrease as similarity parameter η and magnetic field strength parameter M increase. On the other the hand concentration of the fluid is decrease as Schmidt number Sc and Reaction rate parameter δ decrease.

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