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SIMILARITY SOLUTION OF MHD BOUNDARY LAYER FLOW OF PRANDTL-EYRING FLUIDS PAST CORNER OF THE FLAT PLATE

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A B S T R A C T

In the present paper we consider the similarity analysis of magneto-hydrodynamics (MHD) boundary layer flow of non-Newtonian Prandtl-Eyring fluids past corner of the plate placed against free stream velocity. The use two different type group symmetry methods namely: The scaling group symmetry method and the new deductive group analysis are applied to derive possible similarity transformations of the present flow problem. The comparison of both the similarity techniques is also performed. The important conclusion drawn from the present analysis is that for all those non-Newtonian fluids whose shearing stress is composite function of rate of strain, the similarity solutions exist only for the flows past a corner of the plate.

1 Introduction:

Academic curiosity and practical applications have generated considerable interest in finding the solutions of differential equations governing the motion of non-Newtonian fluids. The property of these fluids is that the stress tensor is related to the rate of deformation tensor by some non-linear relationship. From a long, there has been considerable interest in non-

Newtonian fluids. This is because non-Newtonian fluids are found to be of great commercial importance. Examples of such fluids includes slurries, shampoo, toothpaste, paint, clay coating and suspensions, grease, cosmetic products, custard, blood and many others.

It is quite difficult to suggest a single model, which exhibits all properties of non-Newtonian fluids, as mathematical structure of such fluids is not as simple like Newtonian fluids. Further there has been much confusion in the constitutive classification of non-Newtonian fluids. Non-Newtonian fluids are usually classified as : (i) fluids for which shear stress depends only on the rate of shear (ii) fluids for which relation between shear stress and rate of strain depends on time (iii) the viscoinelastic fluids which possess both elastic and viscous properties. Thus for any non-Newtonian fluids the mathematical structure of the shearing stress and the rate of shear is always important. But derivation of such mathematical formulation is indeed a difficult task. Numbers of rheological models have been proposed to explain such a diverse behavior. Some of this models are ; Power-law fluids, Sisko fluids, Ellis fluids, Prandtl fluids Williamson fluids, Sutterby fluids Reiner-Rivlin fluids, Bingham plastic , Prandtl-Eyring fluids , Powell-Eyring fluids , Reiner-Philippoff etc.

The partial differential equations governing the motion of boundary layer flow of non-Newtonian fluid are usually non-linear in nature and hence cannot be solved easily. Whenever possible these differential equations are reduced to ordinary differential equations by some short of transformations known as similarity transformations. These transformations play an important role in mathematical simplification of basic non-linear partial differential equations. To investigate the non-Newtonian effects, the class of solutions known as similarity solutions place an important role. This is because that is the only class of the exact solution for the governing equations which are usually non-linear partial differential equations (PDEs) of the boundary layer type. Further this also serves as a reference to check approximate solutions.

It is well known that similarity solutions for the PDEs governing the flow of Newtonian and non-Newtonian fluids exist only for limited classes of main stream velocities at the edge of the boundary layer. For example, for two dimensional laminar boundary layer flow of Newtonian fluids, similarity solutions are limited to the well known Falkner-Skan solution Rajagopal, Gupta and Na (1983). Most of the generalization of the Falkner-Skan solutions and approximate solutions in the literature are limited to the power law fluids; this is because they are mathematically the easiest to be treated among most of the non-Newtonian fluids.

Hansen and Na (1968) were probably first to derive similarity analysis of laminar incompressible boundary layer equations of the non-Newtonian fluids whose stress and rate of strain are related by arbitrary continuous function, while extending the work of Lee and Ames (1966). They have used linear group of transformations to derive their similarity equations which includes many of the non-Newtonian viscoinelastic fluids. Timol and Kalthia (1986) are probably first to derive similarity solution for the class of three dimensional boundary layer flow of visco-in-elastic non-Newtonian fluids.

Ames (1969), Bluman and Cole (1974), Hansen (1964), Seshadri and Na (1985) and Stephani (1989) have discussed application of groups and symmetries to partial differential equations arising from natural phenomena and technological problems. Symmetry groups are invariant transformations which do not alter the structural form of the equation under investigation. The advantage of the symmetry method is that it can be applied successfully to non-linear partial differential equations governing the motion of fluid. Sophus Lie developed a transformation, currently known as Lie group of transformation, which maps a given differential equation to itself. The differential equations remain invariant under some continuous group of transformations usually known as symmetries of a differential equation.

When we consider electrically conducting non-Newtonian fluids flowing under the influence of external magnetic field, the study becomes interesting .This is because in such situation magnetic forces produced in it could influence the motion of the fluids in significant way and hence such interaction problems have great practical applications. The problem of two-dimensional magneto hydrodynamic boundary layer equation for laminar incompressible flow past flat plate has been investigated by Rossow (1957) and Greenspan and Carrier (1959). Rossow (1957) has considered transverse magnetic field where as Greenspan and Carrier (1959) have considered longitudinal magnetic fields on the velocity and temperature distributions. Timol and Timol (1988) have investigated three-dimensional magneto hydrodynamic boundary layer flow with pressure gradient and fluid injection. Similarity transformation for both steady and unsteady three-dimensional MHD boundary layer flow of purely viscous non-Newtonian fluid has been derived by Patel and Timol (2008). They have also derived Similarity Analysis in MHD Heat and Mass Transfer of Non-Newtonian Power Law Fluids Past a Semi-infinite Flat Plate.

For the derivation of the constitutive equations governing the motion of non-Newtonian fluids, the mathematical structure of stress-strain relationship, which is non linear, is important in when found in functional form. This relationship may be implicit or explicit. In the present paper we have consider such relationship in the form of general arbitrary continuous function of the type

$$\boldsymbol{\Omega\left(\tau_{yx,}\ \frac{\partial u}{\partial y}\right) = 0} \qquad \textbf{(1)}$$

Here τ is the shearing stress and $\frac{\partial u}{\partial y}$ is the rate of the strain of the fluids.

For the similarity analysis many techniques are available, among them the similarity methods which invoke the invariance under the group of transformations are known as group theoretic methods. These methods are more recent and are mathematically elegant and hence they are widely used in different fields. The group theoretic methods involve mainly two different types of groups of transformations, namely, assumed group of transformations, spiral group transformations are the assumed group of transformations and are mainly due to Birkhoff (1960) and Morgan (1952).

In the present paper, the problem of steady two dimensional MHD boundary layer flow of non-Newtonian fluid is analyzed. Prandtl-Eyring model is considered for stress-strain relationship. Similarity solutions are obtained for the present flow problem through two different similarity techniques, namely, Scaling group transformation technique and new deductive group transformation technique [Refer Ateka Pathan (2016)]. It is to be seen that comparison of these two techniques leads to some interesting conclusions.

From the present analysis it is interesting to observe that for non-Newtonian viscoinelastic fluids of any model , which is characterized by the property that its stress and the rate of strain can be related by arbitrary continuous function given by equation (1), the similarity solutions exist only for the flows past corner of the flat plate , as shown in Fig.1

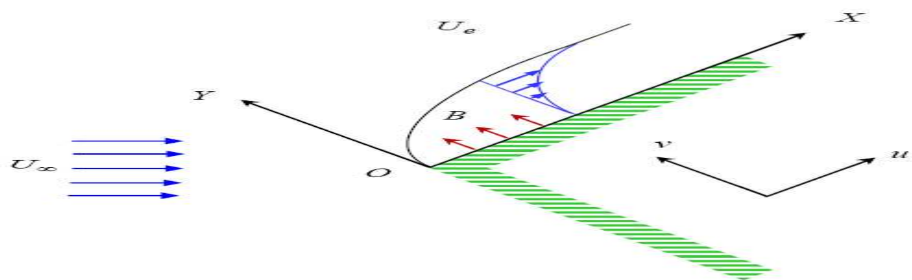

Figure 1: The MHD boundary layer flow past corner of the flat plate.

2 Governing Equation:

As shown in fig.-1, the flow considered here is the two-dimensional boundary layer flow of non-Newtonian fluids the boundary layer flow past a corner of the plate. Under usual boundary layer assumpson the equation of motion for such incompressible electrically conducting non-Newtonian fluid can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\rho}\frac{\partial \tau}{\partial y}+U\frac{\partial U}{\partial x}-\frac{\sigma B_0^2}{\rho}u \tag{3}$$

With stress-strain relationship is given by,

$$\Omega\left(\tau_{yx},\frac{\partial u}{\partial y}\right)=0 \tag{4}$$

Together with boundary conditions,

$$y=0,\ u(x,0)=v(x,0)=0 \tag{5}$$

$$y=\infty, \qquad u(x,\infty)=U(x)$$

The above equation can be made dimenstionless using following quantities,

$$\left\{\begin{array}{c} x^*=\frac{x}{L}\ ,\ y^*=\frac{y}{L}(Re)^{\frac{1}{2}} \\ u^*=\frac{u}{U_\infty},v^*=\frac{v}{U_\infty}(Re)^{\frac{1}{2}} \\ \tau_{yx}^*=\frac{\tau_{yx}}{\rho U_\infty^2}(Re)^{\frac{1}{2}},U^*=\frac{U}{U_\infty} \\ R_e=\frac{U_\infty L}{v},S^*=\frac{L}{U_\infty}\ S \end{array}\right. \tag{6}$$

Where $R_e=\frac{U_\infty L}{v}$ Reynolds number and $S^*(x)=\frac{\sigma B_0^2(x)}{\rho}$ magnetic parameter

Substitute these quantities in equation (1) to (5) and dropping the asterisk, for simplicity

We get,

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \tag{7}$$

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=\frac{\partial}{\partial y^*}\left({\tau^*}_{y^*x^*}\right)+U^*\frac{\partial U^*}{\partial x}-S^*(x)\,u^* \tag{8}$$

With stress-strain relationship is given by,

$$\boldsymbol{\Omega}\left(\boldsymbol{\tau}^*_{yx}, \frac{\partial \mathbf{u}^*}{\partial \mathbf{y}^*}\right) = \mathbf{0} \tag{9}$$

Introducing stream function ψ such that,

$$\mathbf{u}^* = \frac{\partial \psi^*}{\partial \mathbf{y}^*},\ \mathbf{v}^* = -\frac{\partial \psi^*}{\partial \mathbf{x}^*} \tag{10}$$

Equation of continuity (7) gets satisfied identically and Equations (8) and (9) becomes,

$$\frac{\partial \psi^*}{\partial y^*}\frac{\partial^2 \psi^*}{\partial x^* \partial y^*} - \frac{\partial \psi^*}{\partial x^*}\frac{\partial^2 \psi^*}{\partial {y^*}^2} = \frac{\partial}{\partial y^*}\left(\boldsymbol{\tau}^*_{yx}\right) + \mathbf{U}^*\frac{\partial \mathbf{U}^*}{\partial x} - \mathbf{S}^*(\mathbf{x})\ \frac{\partial \psi^*}{\partial y^*} \tag{11}$$

$$\boldsymbol{\Omega}\left(\boldsymbol{\tau}^*{}_{yx}, \frac{\partial^2 \psi^*}{\partial {y^*}^2}\right) = \mathbf{0} \tag{12}$$

With boundary conditions,

$$\begin{cases} \mathbf{y} = \mathbf{0},\ \frac{\partial \psi}{\partial y}(\mathbf{x},\mathbf{0}) = \frac{\partial \psi}{\partial x}(\mathbf{x},\mathbf{0}) = \mathbf{0} \\ \quad \mathbf{y} = \infty, \frac{\partial \psi}{\partial y}(\mathbf{x},\mathbf{y}) = \mathbf{U}(\mathbf{x}) \end{cases} \tag{13}$$

System of equations (11)-(13) is non-linear partial differential equations of boundary value type. We derive similarity solutions of this system by two different similarity techniques: (i) Scaling group transformation technique and (ii) Deductive group transformation technique.

3 Methodology and Solution of the Problem:

First we use scaling linear group transformation:

Introducing following defined ne parameter linear group of transformation:

$$\begin{cases} \bar{\mathbf{x}}^* = \mathbf{A}^{\alpha_1}\mathbf{x}^*,\ \bar{\mathbf{y}}^* = \mathbf{A}^{\alpha_2}\mathbf{y}^* \\ \bar{\psi}^* = \mathbf{A}^{\alpha_3}\psi^*, \bar{\boldsymbol{\tau}}^*_{yx} = \mathbf{A}^{\alpha_4}\boldsymbol{\tau}^*_{yx} \\ \bar{\mathbf{U}}^* = \mathbf{A}^{\alpha_5}\mathbf{U}^*, \bar{\mathbf{S}}^* = \mathbf{A}^{\alpha_6}\mathbf{S}^* \end{cases} \tag{14}$$

Where $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$ and A are Constants

For the dependent and independent variables. From equation (14) one obtains

$$\left(\frac{\bar{x}^*}{x^*}\right)^{\frac{1}{\alpha_1}} = \left(\frac{\bar{y}^*}{y^*}\right)^{\frac{1}{\alpha_2}} = \left(\frac{\bar{\psi}^*}{\psi^*}\right)^{\frac{1}{\alpha_3}} = \left(\frac{\bar{\tau}^*_{yx}}{\tau^*_{yx}}\right)^{\frac{1}{\alpha_4}} = \left(\frac{\bar{U}^*}{U^*}\right)^{\frac{1}{\alpha_5}} = \left(\frac{\bar{S}^*}{S^*}\right)^{\frac{1}{6}} = A \tag{15}$$

Introducing the linear transformation, given by equation (15), into the equations (11-12) results in

$$A^{2\alpha_3-2\alpha_2-\alpha_1}\frac{\partial\bar{\psi}^*}{\partial\bar{y}^*}\frac{\partial^2\bar{\psi}^*}{\partial\bar{x}^*\partial\bar{y}^*} - A^{2\alpha_3-2\alpha_2-\alpha_1}\frac{\partial\bar{\psi}^*}{\partial\bar{x}^*}\frac{\partial^2\bar{\psi}^*}{\partial\bar{y}^{*2}} = A^{\alpha_4-\alpha_2}\frac{\partial}{\partial\bar{y}^*}\left(\bar{\tau}^*_{yx}\right) + A^{2\alpha_5-\alpha_1}\bar{U}^*\frac{\partial U^*}{\partial\bar{x}^*} -$$

$$A^{\alpha_6+\alpha_3-\alpha_2}\frac{\partial\bar{\psi}^*}{\partial\bar{x}^*} = A^{\alpha_4-\alpha_2}\frac{\partial}{\partial\bar{y}^*}\left(\bar{\tau}^*_{yx}\right) + A^{2\alpha_5-\alpha_1}\bar{U}^*\frac{\partial U^*}{\partial\bar{x}^*} - A^{\alpha_6+\alpha_3-\alpha_2}\frac{\partial\bar{\psi}^*}{\partial\bar{x}^*} \tag{16}$$

And

$$\Omega\left(A^{\alpha_4}\tau^*{}_{yx}, A^{\alpha_3-2\alpha_2}\frac{\partial^2\psi^*}{\partial y^{*2}}\right) = 0 \tag{17}$$

The differential equation are completely invariant to the proposed linear transformation, the following coupled algebraic equations are obtained

$$2\alpha_3 - 2\alpha_2 - \alpha_1 = \alpha_4 - \alpha_2 = 2\alpha_5 - \alpha_1 = \alpha_6 + \alpha_3 - \alpha_2 \tag{18}$$

$$\alpha_3 - 2\alpha_2 = 0 \tag{19}$$

$$\alpha_4 = 0 \tag{20}$$

By solving above equations with $\alpha_1 \neq 0$, we get

$$\frac{\alpha_2}{\alpha_1} = \frac{1}{3}\,,\ \frac{\alpha_3}{\alpha_1} = \frac{2}{3}\,,\ \frac{\alpha_4}{\alpha_1} = 0\,,\ \frac{\alpha_5}{\alpha_1} = \frac{1}{3}\,,\ \frac{\alpha_6}{\alpha_1} = -\frac{2}{3} \tag{21}$$

Introducing equation (21) into equation (15) result in

$$\begin{cases} \eta = \dfrac{y^*}{x^{*\frac{1}{3}}}\ ,\ \psi^* = f(\eta)x^{*\frac{2}{3}},\ U^* = G(\eta)x^{*-\frac{1}{3}} \\ \tau^*_{yx} = H(\eta) \quad \text{and}\ S(x) = S_0 x^{*-\frac{2}{3}} \end{cases} \tag{22}$$

With the boundary conditions, equation (13) becomes

$$\begin{cases} \eta = 0,\ f(0) = f'(0) = 0 \\ \eta \to \infty, f'(\infty) = 1 \end{cases} \tag{23}$$

Introducing equations (22) in equation (11)-(13), we get following similarity equation

$$f'^2(\eta) - 2f(\eta)f''(\eta) - 3H'(\eta) + \eta G(\eta)\,G'(\eta) - G^2(\eta) + S_0\big(f'(\eta)\big) = 0 \tag{24}$$

But $U^{*}$is independent of y, $G(\eta)$ must be constant.

Therefore $G(\eta)$ assume Unity i.e. $G(\eta) = 1$, $G'(\eta) = 0$

$$\mathbf{f'^2(\eta) - 2f(\eta)f''(\eta) - 3H'(\eta) - 1 + S_0\big(f'(\eta)\big) = 0} \tag{25}$$

With the boundary conditions,

$$\begin{cases} \boldsymbol{\eta = 0\,,\ f(0) = f'(0) = 0} \\ \qquad \boldsymbol{\eta \to \infty, f'(\infty) = 1} \end{cases} \tag{26}$$

And the Stress-Strain functional relationship is given by,

$$\boldsymbol{\Omega(H, f'') = 0} \tag{27}$$

3.4 Similarity solution by Group Systematic Invariance Analysis (New Deductive Group method)

Dropping the asterisks (for simplicity) and using $S(x) = bB^2(x)$, in equation (11), we get

$$\boldsymbol{\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} = \frac{\partial\tau_{yx}}{\partial y} + U\frac{dU}{dx} - bB^2(x)\ \frac{\partial\psi}{\partial y}}\ , \tag{28}$$

Subject to related boundary conditions:

$$\boldsymbol{\frac{\partial\psi}{\partial y}(x,0) = \frac{\partial\psi}{\partial x}\ (x,0) = 0,\ \frac{\partial\psi}{\partial y}(x,\infty) = U(x).}$$

Equation (28), contains two independent variables. Hence to transform it into ordinary differential equation, as shown in chapter-2, we initiated the procedure with the group C_G, a class of transformation of one-parameter $'\varepsilon'$ of the form:

$$\boldsymbol{C_G: \bar{s} = D^s(\varepsilon)s + T^s(\varepsilon)} \tag{29}$$

Where s stands for $x, y, \psi, U, \tau_{yx}, B$ whereas $D's\ and\ T's$ are real-valued and are at least differentiable in the real argument ε.

To transform the differential equation (28), transformations of the derivatives of ψ are obtained from C_G via chain-rule operations:

$$\left.\begin{aligned}\bar{s}_{\bar{i}} &= \left(\frac{D^s}{D^i}\right) s_i \\ \bar{s}_{\bar{i}\bar{j}} &= \left(\frac{D^s}{D^i D^j}\right) s_{ij}\end{aligned}\right\} s = \psi, U, \tau_{yx}, B; i, j = x, y \tag{30}$$

Here suffixes denote partial derivatives

Equation (28) is said to be invariantly transformed, for some function $\chi(\varepsilon)$ whenever,

$$\frac{\partial\bar{\psi}}{\partial\bar{y}}\frac{\partial^2\bar{\psi}}{\partial\bar{y}\partial\bar{x}} - \frac{\partial\bar{\psi}}{\partial\bar{x}}\frac{\partial^2\bar{\psi}}{\partial\bar{y}^2} - \frac{\partial\bar{\tau}_{\bar{y}\bar{x}}}{\partial\bar{y}} - \bar{U}\frac{d\bar{U}}{d\bar{x}} + b\bar{B}^2(\bar{x})\frac{\partial\bar{\psi}}{\partial\bar{y}}$$
$$= \chi(\varepsilon)\left[\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} - \frac{\partial\tau_{yx}}{\partial y} - U\frac{dU}{dx} + bB^2(x)\frac{\partial\psi}{\partial y}\right]$$

Substituting the values from the equations (29) and (30) in above equation, yields

$$\frac{(D^\psi)^2}{D^x(D^y)^2}\left[\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\right] - \frac{D^{\tau_{yx}}}{D^y}\frac{\partial\tau_{yx}}{\partial y} - (D^U U + T^U)\frac{D^U}{D^x}\frac{dU}{dx} + b(D^B B + T^B)^2\frac{D^\psi}{D^y}\frac{\partial\psi}{\partial y} =$$
$$\chi(\varepsilon)\left[\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} - \frac{\partial\tau_{yx}}{\partial y} - U\frac{dU}{dx} + bB^2(x)\frac{\partial\psi}{\partial y}\right] \tag{31}$$

The invariance of equation (31) together with boundary conditions (28), implies that

$$\begin{cases} T^U = T^{\tau_{yx}} = T^y = T^\psi = T^B = 0 \\ \frac{(D^\psi)^2}{D^x(D^y)^2} = \frac{D^{\tau_{yx}}}{D^y} = \frac{(D^U)^2}{D^x} = \frac{(D^B)^2 D^\psi}{D^y} = \chi(\varepsilon)\end{cases} \tag{32}$$

These yields,

$$D^x = (D^y)^3, D^\psi = (D^y)^2, D^U = D^y, D^{\tau_{yx}} = 1,\ D^B = \frac{1}{D^y} \tag{33}$$

The one-parameter sub-group G of C_G, which transforms invariantly the governing equations with the auxiliary conditions is

$$G: \begin{cases} S: \begin{cases}\bar{x} = (D^y)^3 x + T^x, \\ \bar{y} = D^y y\end{cases} \\ \bar{\psi} = (D^y)^2\psi,\ \bar{U} = D^y U, \bar{\tau}_{\bar{y}\bar{x}} = \tau_{yx},\ \bar{B} = \frac{1}{D^y}B\end{cases} \tag{34}$$

The Complete Set of Absolute Invariants:

For a one-parameter group, if $\eta = \eta(x, y)$ is the absolute invariant of the independent variables then, the four absolute invariants of for the dependent variables ψ, U, B, τ_{yx} are given by

$$F_j(x, y, \psi, U, \tau_{yx}, B) = f_j(\eta), \ \ j = 1, 2, 3, 4 \tag{35}$$

And can be obtained by following first-order linear partial differential equation: (see Morgan (1952),.Moran and Gaggioli (1968) , Eisenhart(1961))

$$\sum_{i=1}^{6}(\alpha_i s_i + \beta_i)\frac{\partial F}{\partial s_i} = 0, \ s_i = x, y, \psi, U, \tau_{yx}, B \tag{36}$$

Where

$$\alpha_i = \left.\frac{\partial D^i}{\partial \varepsilon}\right|_{\varepsilon=\varepsilon^0} \ and \ \ \beta_i = \left.\frac{\partial T^i}{\partial \varepsilon}\right|_{\varepsilon=\varepsilon^0} \ \ i = 1, 2, 3, 4, 5, 6 \tag{37}$$

And ε^0 denotes the value of parameter ε which yields the identity element of the group G. The absolute invariant of independent variables owing the equation (36) is $\eta = \eta(x, y)$ if it will satisfy the first order linear partial differential equation

$$(\alpha_1 x + \beta_1)\frac{\partial \eta}{\partial x} + (\alpha_2 y + \beta_2)\frac{\partial \eta}{\partial y} = 0 \tag{38}$$

Using the definition of $\alpha's \ \ and \ \beta's$

$$(x + \beta)\frac{\partial \eta}{\partial x} + \frac{y}{3}\frac{\partial \eta}{\partial y} = 0, \ where \ \ \beta = \frac{\beta_1}{3\alpha_1} \tag{39}$$

The characteristics equation of (39) is

$$\frac{dx}{(x+\beta)} = \frac{3dy}{y} = \frac{d\eta}{0} \tag{40}$$

Applying the variable separable method the absolute invariant of independent variables is

$$\eta(x, y) = y(x + \beta)^{-1/3} \tag{41}$$

Similarly the absolute invariants for dependent variables owing the equation (36), one can derive, are:

$$\left.\begin{array}{ll} f_1(\eta) = \psi(x + \beta)^{-2/3}, & f_2(\eta) = U(x + \beta)^{-1/3} \\ f_3(\eta) = \tau_{yx} \qquad , & f_4(\eta) = B(x + \beta)^{1/3} \end{array}\right\} \tag{42}$$

Since $U(x) and \ B(x)$are functions of x only, $f_2(\eta)$ and $f_4(\eta)$ must be constants say $U_0 \ and \ B_0$ respectively.

The group transformation of absolute invariants is

$$\left.\begin{array}{ll} \boldsymbol{\psi = (x+\beta)^{2/3} f(\eta)}, & \boldsymbol{U = U_0 (x+\beta)^{1/3}} \\ \boldsymbol{\tau_{yx} = g(\eta)} \quad , & \boldsymbol{B = B_0 (x+\beta)^{-1/3}} \end{array}\right\} \tag{43}$$

Where $f(\eta) = f_1(\eta)$ and $g(\eta) = f_3(\eta)$.

Reduction to Ordinary Differential Equation:

Substituting the values of derivatives from (43) in equation (28), one can reduce the following differential equation:

$$\boldsymbol{(f')^2 - 2ff'' - 3g' - {U_0}^2 + 3M_n f' = 0} \tag{44}$$

Where $M_n = \sigma {B_0}^2 / \rho U_\infty$ is the magnetic parameter and $g(\eta)$ is similarity variable related to non-dimensional strain-stress relation (1), whence

$$\boldsymbol{g'^{(\eta)} = \frac{\alpha f'''}{\{1 + \gamma (f'')^2\}^{1/2}}}\ , \qquad \boldsymbol{where\ \alpha = \frac{A}{\mu B},\ \ \gamma = \frac{\rho {U_\infty}^3}{\mu L B^2}}$$

Substituting the value in equation (44), we get

$$\boldsymbol{f''' = \frac{1}{3\alpha} \{(f')^2 - 2ff'' - {U_0}^2 + 3M_n f'\} \{1 + \gamma (f'')^2\}^{1/2}} \tag{45}$$

In which α and γ are dimensionless number and are referred as flow parameters and primes denote derivative with respect to similarity variable η.

The boundary conditions (28) transform to

$$\boldsymbol{f(0) = f'(0) = 0,\ f'(\infty) = U_0} \tag{46}$$

Further the expression of local skin-friction coefficient C_f is

$$\boldsymbol{\frac{1}{2}\sqrt{R_e} C_f \equiv \tau_{yx}\Big|_{y=0} = \frac{\alpha}{\sqrt{\gamma}} sinh^{-1}\{\sqrt{\gamma} f''(0)\}}$$

Comparison:

Equation (44) is identical to equation (25) for $U_0 = 1,\ 3M_n = S_0$, are all constants. Where boundary conditions remains same.

Conclusion:

- The similarity solutions for MHD laminar incompressible boundary layer equations of all non-Newtonian Prandtl-Eyring fluids are derived.
- It is interesting to note that the deductive group theoretic method based on general group of transformation is applied to derive proper similarity transformations for the non-linear partial differential equation with the stress-strain functional relationship condition, governing the flow under consideration.
- It is to be observing that similarity solutions for all non-Newtonian fluids exist only for the flow past corner of the flat plate.
- It is worth to note that all solutions have derived in terms of non-dimensional form and hence these results can be applicable to all non-Newtonian fluids.

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