ISSN : 2320-2971 (Online)

International e-Journal for Education and Mathematics

www.iejem.org Vol. 02, No. 04, (Aug. 2013), pp 21-36

Models Of Various Non-Newtonian Fluids

M Patel[1], H Surati[2], M. S. Chanda[3] and M.G.Timol[4]

[1,2]Department of Mathematics, Sarvajanik College of Engineering & Technology, Surat-395001, Gujarat, INDIA, E-mail: 1. manishapramitpatel@gmail.com, 2. hema.surati@scet.ac.in

[3] Department of Mathematics, Poona College for Arts Science and Commerce, Pune, INDIA
E-mail: sadiquechanda@yahoo.com

[4] Department of Mathematics, Veer Narmad South Gujarat University,
Magdalla Road, Surat-395007, Gujarat, INDIA, E-mail: mgtimol@gmail.com

Article Info.

Received on 03 July 2013
Revised on 22 July 2013
Accepted on 22 July 2013

Keywords:
non-Newtonian fluids, Stress-Strain relationship, Deformation

ABSTRACT

The classical theory of the fluid dynamics of viscous fluids depends on the relations between the components of stress in a fluid and those of the strain velocity. Here the most general possible relations between the stress and strain velocity components, which can be obeyed by incompressible, visco-inelastic non-Newtonian fluids, are given. Different non-Newtonian fluid models are classified according to their mathematical functional relationship.

AMS Subject classification: *76A05, 76M55, 54H15*

1. Introduction

The complex rehology of biological fluids has motivated investigations involving different non-Newtonian fluids. In recent years, non-Newtonian fluids have become more and more important industrially. Academic curiosity and practical applications have generated considerable interest in finding the solutions of differential equations governing the motion of non-Newtonian fluids. The property of these fluids is that the stress tensor is related to the rate of deformation tensor by some non-linear relationship. These fluids present some interesting challenges to researchers in engineering, applied mathematics and computer science. Many materials such as drilling mud, clay coating and other suspensions, certain oils and greases, polymer melts, blood, paints and certain oils, elastomers and many emulsions and some other thin and thick oils have been treated as non-Newtonian fluids. Because of the difficulty to suggest a single model, which exhibits all properties of non-Newtonian fluids, they cannot be described simply as Newtonian fluids and there has been much confusion in the classification of non-Newtonian fluids. Non-Newtonian fluids are usually classified as : (i) fluids for which shear stress depends only on the rate of shear (ii) fluids for which relation between shear stress and rate of strain depends on time (iii) the viscoinelastic fluids which possess both elastic and viscous properties. Thus for any non-Newtonian fluids the mathematical structure of the shearing stress and the rate of shear is always important. But such a mathematical formulation is indeed a difficult task. It is interesting to note that a very nice work on non-Newtonian fluid along with constitutive stress-strain relationship has been discussed by Wilkinson (1960) and Kapur (1982) in their text. Also stress-strain relationship for 3-D Cartesian coordinate system for many non-Newtonian models has been derived by Timol (1986). Recently, a most important article of stress strain relationship for viscous-inelastic non-Newtonian fluids has been published by Patel and Timol (2010). They have made a detail analysis for some important non-Newtonian fluids.

Understanding the flow phenomena along with the corresponding environmental changes (pressure drop, concentration gradient, etc.) becomes important in many fields, such as liquid transport through geo-membranes, effluent movement through filtration devices, and chemical movement through protective apparel. The difference between Newtonian and non-Newtonian fluids and how they differ in their resistance to forced movement through a fabric is explained by Matthew W. Dunn (Dunn, 1999) using the stress-strain relationship. Indeed a difficult task. This is because there is a great diversity found in the physical structure of non-Newtonian fluids and hence it is quite difficult to recommend a single constitutive equation which can be use to describe all above three classifications. For this reason many non-Newtonian fluid model for constitutive equation have been proposed and most of them are empirical or semi-empirical. The present research on the flow problem for non-Newtonian fluid is hampered by the lacking of proper classification of mathematical structure of a stress-strain relationship. The attempt has been made to explain different models of Non-Newtonian fluids with stress-strain relationship.

Shear Stress versus Shear Rate:

Most common fluids such as water, air and gasoline are Newtonian under normal conditions. Non-Newtonian fluids occur commonly in our world. These fluids, such as toothpaste, saliva, oils, mud and lava, exhibit a number of behaviors that are different form Newtonian fluids and have a number of additional material properties. In general, these differences arise because the fluid has a microstructure that influences the flow.

A plot of shear stress versus shear rate at a given temperature is a straight line with a constant slope that is independent of the shear rate. We call this slope the viscosity of the fluid. All gases are Newtonian. Also, law molecular weight liquids and solutions of low molecular weight substances in liquids are usually Newtonian. Some examples are aqueous solution of sugar or salt.

Any fluid that does not obey the Newtonian relationship between the shear stress and shear rate is called non-Newtonian. The subject of "Rehology" is devoted to the study of the behavior of such fluids. High molecular weight liquids which include polymer melts and solutions of polymers, as well as liquids in which fine particles are suspended (slurries and pastes), are usually non-Newtonian. In this case, the slope of the shear stress versus shear rate curve will not be constant as we change the shear rate. When the viscosity decreases with increasing shear rate, we call the fluid shear-thinning. In the opposite case where the viscosity increase as the fluid is subjected to a higher shear rate, the fluid is called shear-thickening (dilatants). Shear- thinning behavior is more common than shear-thickening. Shear–thinning fluids also are called Pseudo plastic fluids. A typical shear stress versus shear rate plot for a shear thinning fluid is given as Figure 1.

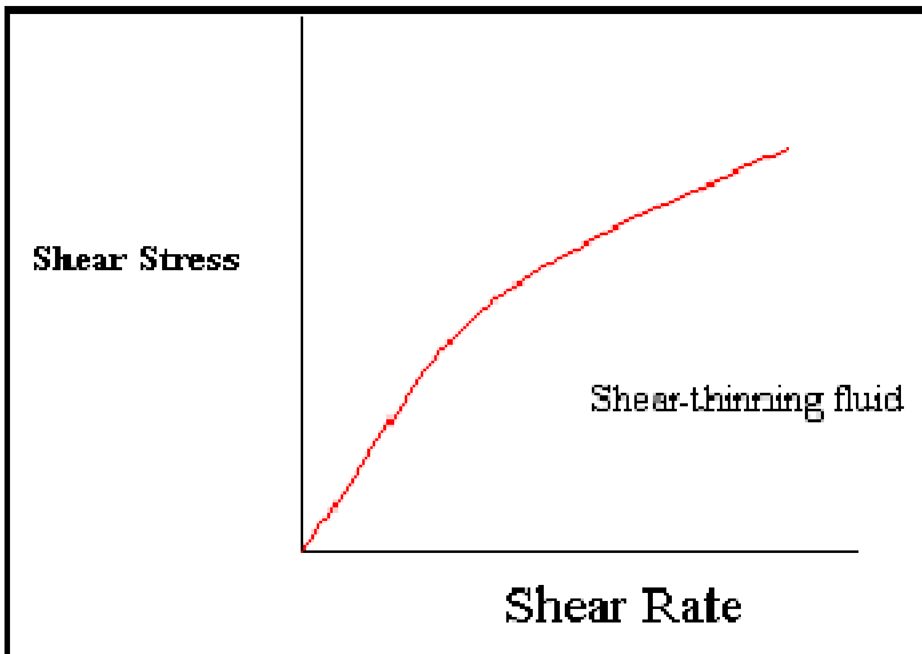

Figure 1: **Graph for Shear-thinning fluid**

We describe the relationship between the shear stress τ and shear rate θ as follows:

$$\tau = \eta\theta$$

where η is called the "apparent viscosity" of the fluid, and is a function of the shear rate. In the above example, a plot of η as a function of the shear rate θ looks like Figure 2. Many shear –thinning fluids will exhibit Newtonian behavior at extreme shear rates, both low and high.

For such fluids, when the apparent viscosity is plotted against log shear rate, we see a curve given as Figure 3.

The region where the apparent viscosity is approximately constant are known as Newtonian region. The behavior between these regions can usually be approximated by a straight line on these axes. It is known as the power-law region. In this region, we can approximate the behavior by $\log \eta = a + b \log \theta$

Which can be rewritten as $\eta = k\theta^b$

Where k = exp (a). Instead of b we commonly use (n – 1) for the exponent and write a result for the apparent viscosity as : $\eta = k\theta^{n-1}$

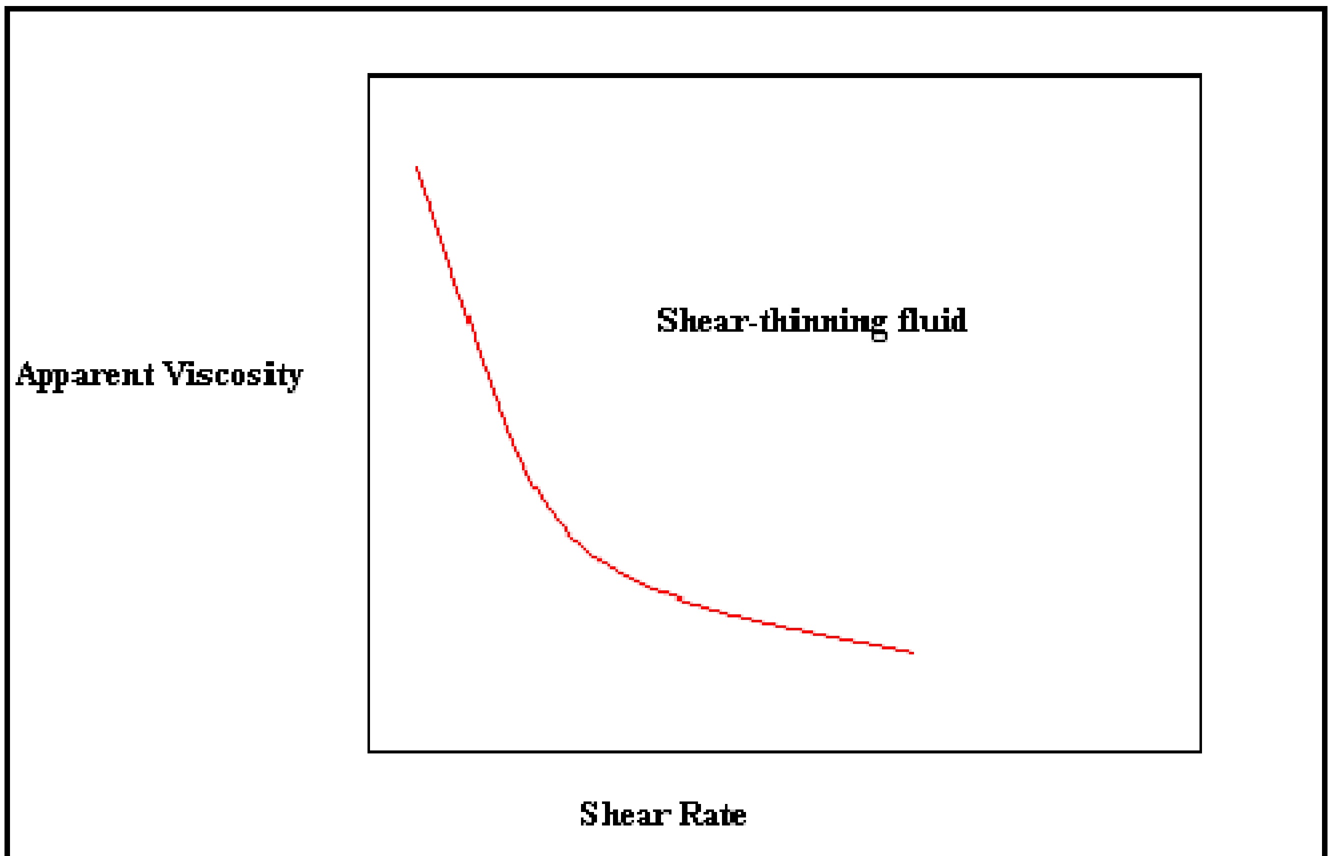

Figure 2: Graph of apparent viscosity vs shear rate

Upon using the connection among the shear stress, apparent viscosity, and the sheer rate are get the power-law model

$$\tau = k\,\theta^{n} \qquad [n < 1 \text{ shear shinning}, \quad n > 1 \text{ shear thickning}]$$

Where n is called the power-law index. Note that n = 1 corresponds to Newtonian behavior. Typically, for shear thinning fluids, n lie between $\frac{1}{3}$ *and* $\frac{1}{2}$, even though other values are possible.

Examples of shear-thinning fluids are polymer melts such as molten polystyrene, polymer solutions such as polyethylene oxide in water and some paints. You can see that when paint is sheared with a brush, it flows comfortably, but when the shear stress is removed, its viscosity increases so that if no longer flows easily. Of course the solvent evaporates soon and then the point sticks to the surface. The behavior of paint is a bit more complex than this, because the viscosity changes with time at a given shear rate.

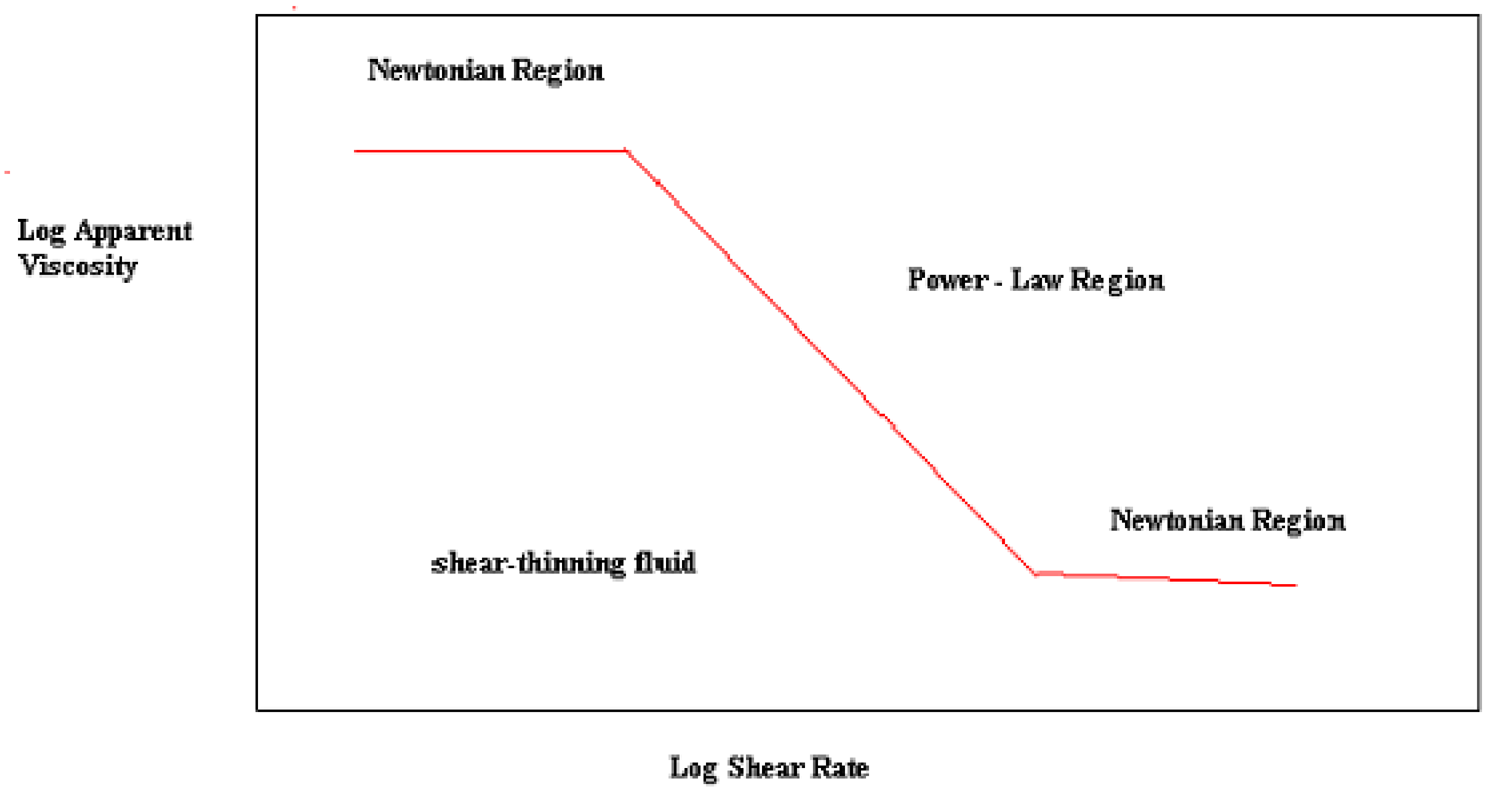

Figure 3: Graph of log apparent viscosity vs log shear rate

Some slurries and pastes exhibit an increase in apparent viscosity as the shear rate is increased. They are called shear thickening or dilatants fluids. Typical plots of shear stress versus shear rate and apparent viscosity versus shear rate are shown as Figure 4.

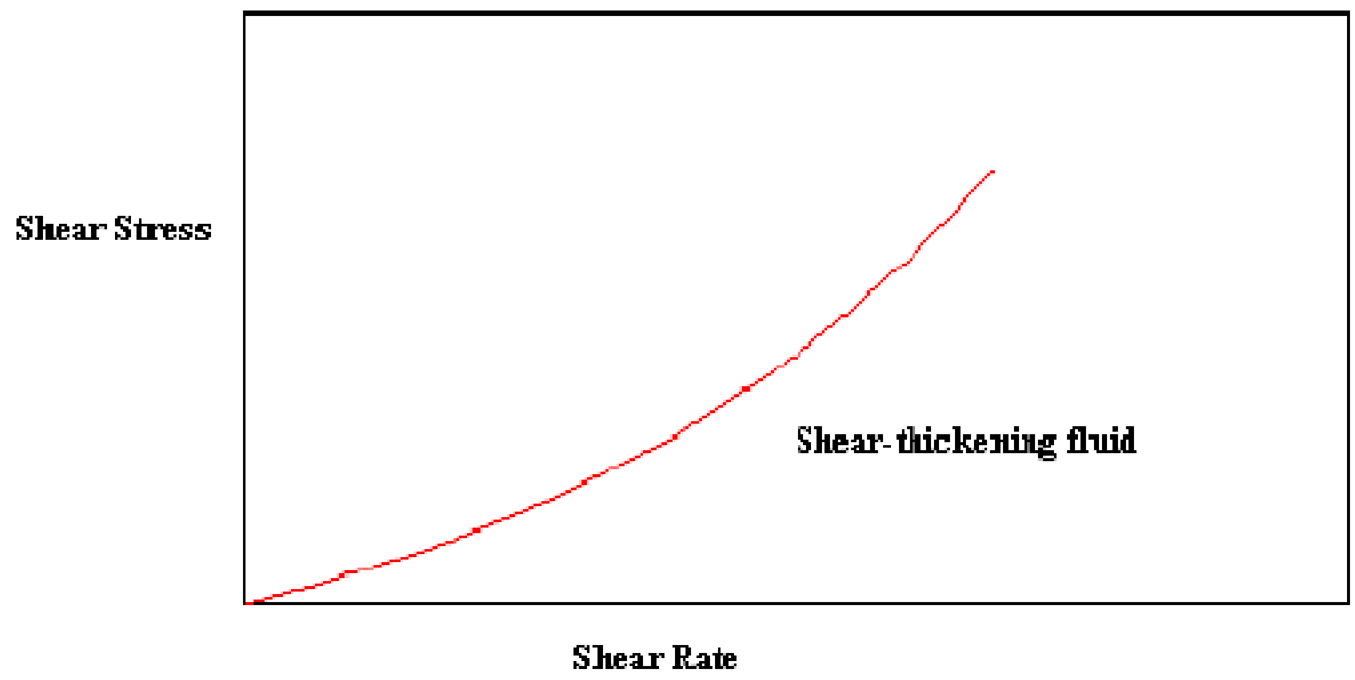

Figure 4: Graph of Shear stress vs shear rate

Some examples of shear thickening fluids are corn starch, clay slurries and solutions of certain surfactants. Most shear-thickening fluids tend to show shear-thinning at very low shear rates. When the apparent viscosity is plotted against log shear rate, we see a curve given as Figure 5.

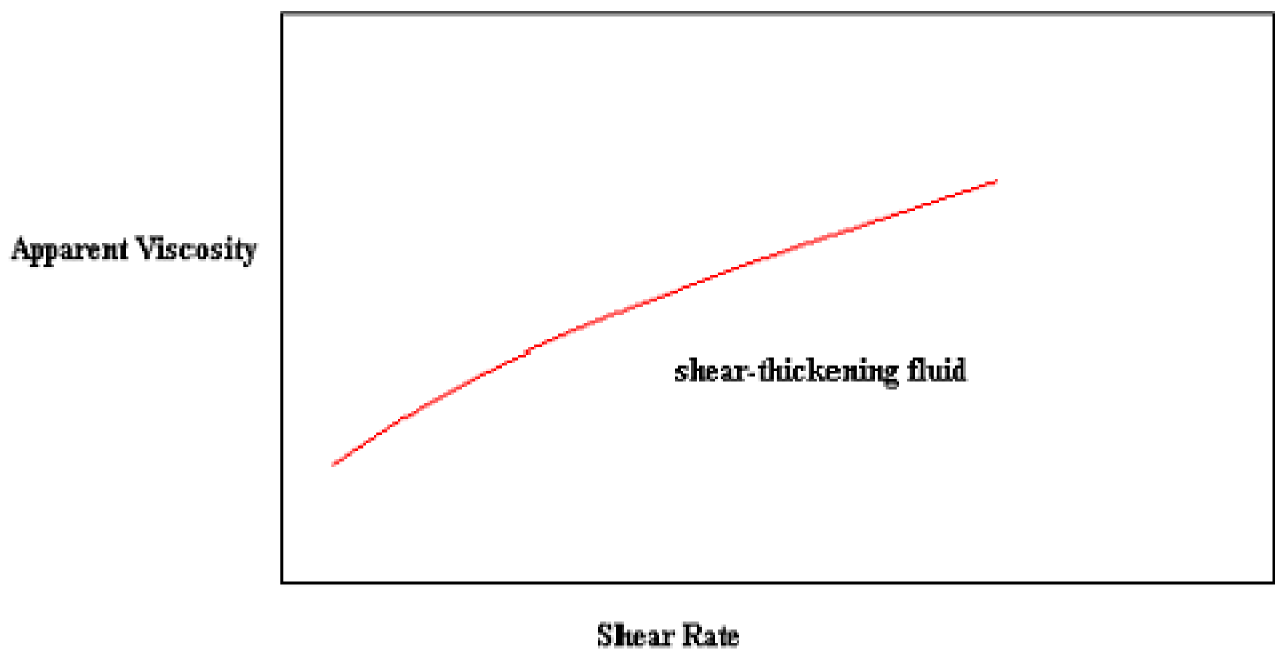

Figure 5: Graph of apparent viscosity vs shear rate

Another important type of non-Newtonian fluids is a viscoplastic or "yield stress" fluid. This is a fluid which will not flow when only a small shear stress is applied. The shear stress must exceed a critical value known as the yield stress τ_0 for the fluid to flow. For example, when you open a tube of toothpaste, it would be good if the paste does not flow at the slightest amount of shear stress. We need to apply an adequate force before the toothpaste will start flowing. So, viscoplastic fluids behave like solids when the applied shear stress is less than the yield stress. Once it exceeds the yield stress, the viscoplastic fluid will flow just like a fluid. Bingham plastics are a special class of viscoplastic fluids that exhibit a linear behavior of shear stress against shear rate. Typical viscoplastic behaviors are illustrated in figure 6.

Examples of viscoplastic fluids are drilling mud, nuclear fuel slurries, mayonnaise, toothpaste and blood. Also, some paints exhibit a yield stress.

Of course, this is not an exhaustive discussion of non-Newtonian behaviors. For instance, some class of fluids exhibit time dependent behavior. This means that even under a given constant shear rate, the viscosity may vary with time. The viscosity of a thixotropic liquid will decrease with time under a constant applied shear stress. However, when the stress is removed, the viscosity will gradually recover with time as well. Non-drip points behave in this way. The opposite behavior, where in the fluid increases in viscosity with time when a constant shear stress is applied, is not as common, and such a fluid is called a rheopectic fluid.

Another important class of fluids exhibits viscoelastic behavior. This means that these fluids bear both of solid (elastic) and fluids (viscous) properties. Viscoelastic fluids exhibit strange phenomena such as climbing up a rotating shaft, swelling when extruded out of a dye etc. An example of a common viscoelastic liquid is egg-white.

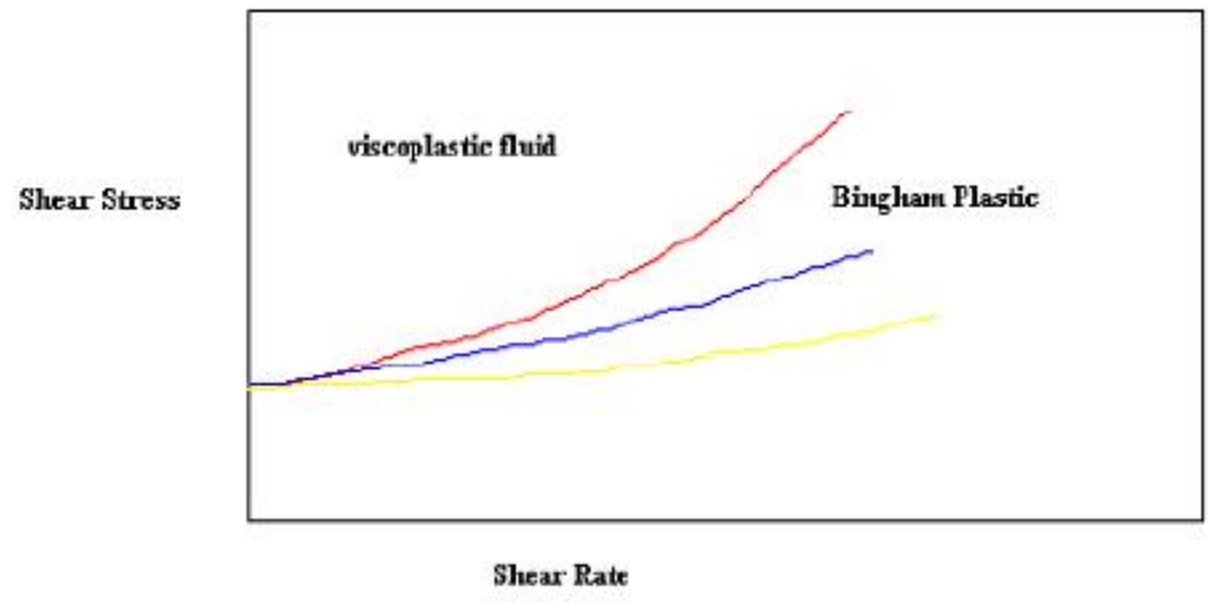

Figure 6: Viscoplastic behaviors.

You have probably noticed that when it flows out of a container, you can use a quick jerking motion to snap it back into the container. Several industrially important polymers melt and solutions are viscoelastic. Shear stress versus Deformation rate is presented by Figure 7.

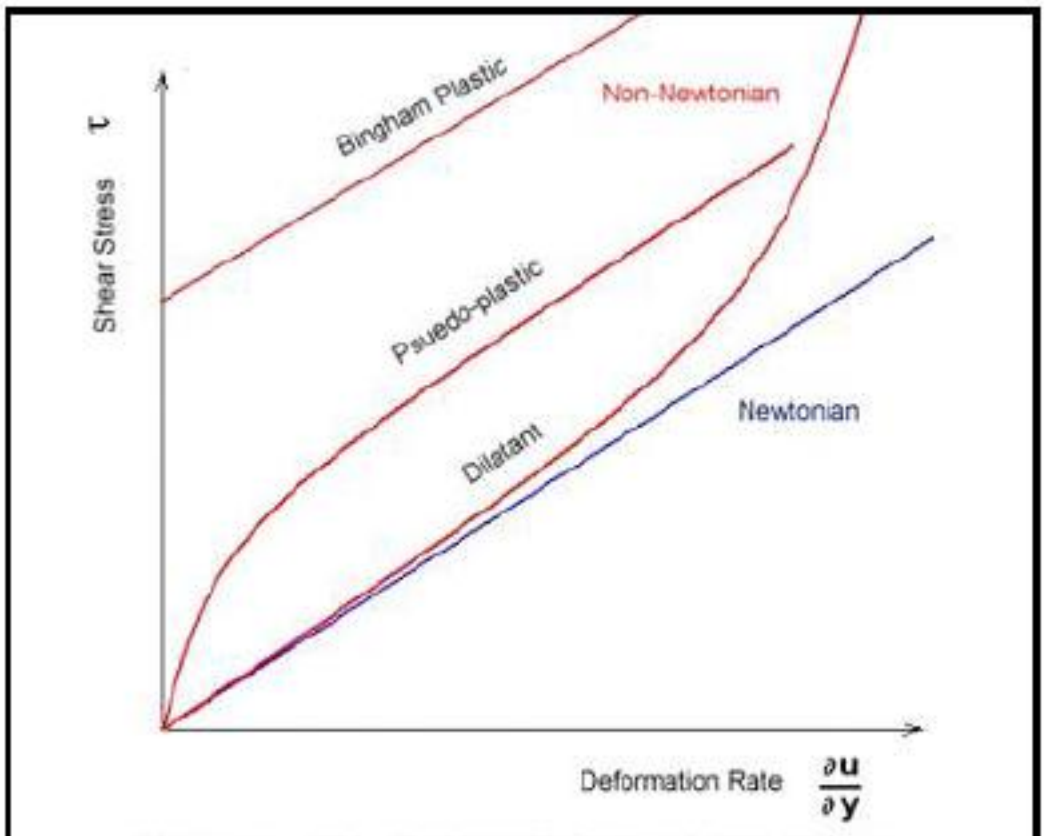

Figure 7: Shear stress vs deformation rate

We have in definition of non-Newtonian fluids;

$\tau_{yx} = k\left(\frac{du}{dy}\right)^n$ This equation reduces to Newton's low of viscosity for n = 1 & k = μ .

VISCO – INELASTIC FLUIDS:

A common feature of this class of fluids is that when at rest they are isotropic and homogenous and when they are subjected to a shear, the resultant stress depends only on the rate of shear. However, this sub-class shows diverse behavior in response to applied stress. A number of rheological models have been proposed to explain such a diverse behavior. Some of these models which have attracted researchers are:

1. Power law fluids: These fluids are characterized by the rheological equation

$$\tau_{ij} = k \left| \left(\sum_{m=1}^{3} \sum_{l=1}^{3} e_{ml} \, e_{lm} \right)^{\frac{1}{2}} \right|^{n-1} e_{ij}$$

where k and n are called the consistency and flow behavior indices respectively. If n < 1, the fluid is called pseudo plastic power law fluid and if n > 1, it is called dilatants power law fluid since the apparent viscosity decreases or increases with the increase shear of rate according as n < 1 or n > 1.

Values of the different parameters of power law fluids have been given by A. B. Metzner (1956).

2. Reiner – Rivlin fluids: Reiner and Rivlin established that for an isotropic fluid, the most general relation between the stress tensor t_{ij} and the rate of deformation tensor e_{ij} has the form

$$t_{ij} = -p\,\delta_{ij} + 2\mu\, e_{ij} + 2\mu_c\, e_{ik}\, e_{kj}$$

Where

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

p is an arbitrary hydrostatic pressure and the coefficient of viscosity μ and coefficient of cross-viscosity μ_c depend on the invariants I_1, I_2 and I_3 where

$$I_1 = e_{ii},\ I_2 = \frac{1}{2} e_{ij}\, e_{ji},\ I_3 = e_{ij}\, e_{jk}\, e_{ki}$$

and where the summation convention is used.

Another general relation between the stress tensor matrix $[t_{ij}]$ and the rate of deformation tensor matrix $[e_{ij}]$ which has been used is

$$[t_{ij}] = -p[\delta_{ij}] + \mu_1[e_{ij}] + \mu_2[e_{ij}]^2 + \mu_3[e_{ij}]^3 + ..$$

where $\mu_1, \mu_2, \mu_3, ...$ are constants.

Particular cases of the first equation has been studied by choosing $\mu_c = 0$ and by assuming a particular form for $\mu = \mu(I_2, I_3)$.

3. Bingham plastics: The empirical relation for the plastic flow of an isotropic fluid is described as $\tau = \pm\tau_0 + \mu e$

Where τ_0 is the yield value, a quantity equal to zero in Newtonian fluid. Following relation has been used by Oldroyd (1947)

$$eij = \begin{cases} \frac{1}{\mu}\frac{\sqrt{(J_2)}-\tau_0}{\sqrt{(J_2)}}\tau_{ij}, \sqrt{(J_2)} \geq \tau_0 \\ 0, \quad , \quad \sqrt{(J_2)} < \tau_0, \end{cases}$$

Where $J_2 = \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3} S_{ij}\, S_{ji}$, and S_{ij} are the components of the deviatoric stress tensor.

4. Ellis Fluids: The rheological relation between the stress tensor S_{ij} and the strain rate tensor is given in the form

$$S_{ij} = \tau_{ij} - pg_{ij},$$

$$e_{ij} = \phi_0\, \tau_{ij} + \phi_1 \left| \left(\sum_{l=1}^{3}\sum_{m=1}^{3} \tau_{lm}\, \tau_{ml} \right)^{\frac{1}{2}} \right|^{\alpha-1} \tau_{ij}$$

Where p is pressure, g_{ij} is the matrix tensor, ϕ_0, ϕ_1 *and* α are the fluid parameters. This fluid exhibits the following interesting properties:

(i) $\phi_1 = 0$, it behaves as a Newtonian fluid,

(ii) $\phi_0 = 0$, it gives a power law model,

(iii) $\alpha > 1$, and if stress components are small, it approximates to Newtonian fluid,

(iv) $\alpha < 1$, and if stress components are large, it approximates to Newtonian fluid,

(v) $\alpha = 1$, it represents Newtonian fluid.

The value of the Ellis fluid parameters for solutions of Carboxymethylcellulose in water has been given by J. C. Slattery (1959).

5. Reiner-Philippoff fluid: The rheological relation between the stress tensor τ_{ij} and the strain rate tensor e_{ij} is given as

$$\tau_{ij} = \left[\mu_0 + \frac{\mu_\infty - \mu_0}{1 + \left(\tau_{lm}\, \tau_{ml} / 2\tau_0^{\,2} \right)} \right] e_{ij},$$

Where μ_0, μ_∞ *and* τ_0 are the fluid parameters. Usual summation convention over values 1, 2, 3 for the repeated indices has been assumed. In the limiting cases, when the parameter τ_0 tends to zero or infinity, the constitutive equation reduces to that of Newtonian fluid with viscosity μ_0 *or* μ_∞.

The values of the Reiner-Philippoff parameters for various fluids been given by W. Philippoff (1935).

6. Prandtl fluids: The empirical relation for the above class of fluids is

$$\tau = A\sin^{-1}\left(\frac{e}{C}\right),$$

where A and C are the material constants of the fluid. A possible generalization is

$$\tau_{ij} = \frac{A\sin^{-1}\left[\left(\sum_{l=1}^{3}\sum_{m=1}^{3}\frac{e_{lm}e_{ml}}{2C^2}\right)^{\frac{1}{2}}\right]}{\left(\sum_{l=1}^{3}\sum_{m=1}^{3}\frac{e_{lm}e_{ml}}{2}\right)^{\frac{1}{2}}}e_{ij}$$

7. Eyring fluids: These are characterized by the following empirical relation

$$\tau = \frac{e}{B} + C\sin\left(\frac{\tau}{A}\right),$$

where A, B and C are the fluid parameters. A possible generalization of above equation is

$$\tau_{ij} = \frac{1}{B}e_{ij} + \frac{C\sin\left[\left(\sum_{l=1}^{3}\sum_{m=1}^{3}\frac{\tau_{lm}\tau_{ml}}{2A^2}\right)^{\frac{1}{2}}\right]}{\left(\sum_{l=1}^{3}\sum_{m=1}^{3}\frac{\tau_{lm}\tau_{ml}}{2}\right)^{\frac{1}{2}}}\tau_{ij}.$$

8. Powell – Eyring fluids: These fluids are characterized by the relation

$$\tau = Ae + B\sinh^{-1}(Ce),$$

Which are generalized to

$$\tau_{ij} = Ae_{ij} + B\sinh^{-1}\left[C\sqrt{\left(\tfrac{1}{2}I_2\right)}\right]\frac{e_{ij}}{\sqrt{\left(\tfrac{1}{2}I_2\right)}},$$

Where A, B and C are the fluid parameters.

9. Williamson fluids: These fluids are governed by the following relation

$$\tau = \frac{Ae}{B+e} + \mu_\infty e$$

The above relation may also be put as

$$e = \alpha\tau + \beta + \sqrt{\left[(\alpha e + \beta) + \gamma\right]},$$

Where α, β *and* γ are the parameters of the fluid. A possible generalization of first equation is,

$$\tau_{ij} = \frac{Ae_{ij}}{B + \left(\sum_{l=1}^{3} \sum_{m=1}^{3} \frac{e_{ml}\ e_{lm}}{2} \right)^{\frac{1}{2}}} + \mu_{\infty} e_{ij}$$

10. Rabinowitsch type fluids: The empirical relation for such fluids is

$$e = \frac{1}{\mu_0}\tau + \sum B_q\, \tau^{2q+1}$$

This is generalized to

$$e_{ij} = \frac{1}{\mu_0}\tau_{ij} + \sum B_q\, J_2^q \tau_{ij}$$

11. Meter - Model: The constitutive equation for this class of fluids is

$$\tau = -\eta \Delta$$

τ *and* Δ being usual shear stress tensor and rate of deformation tensor respectively,

$$\eta = \eta_0 + \frac{\eta_0 - \eta_{\infty}}{1 + \left| \tau / \tau_m \right|^{\alpha - 1}},$$

Where η_0, η_{∞} *and* τ_m are fluid parameters whose values have been tabulated by Meter and Bird (1964). If may further be observed that for $\alpha = 2$, the equation reduce to Peek – Mclean model, and for $\alpha = 3$, it reduce to Reiner – Philippoff model.

VISCO – ELASTIC FLUIDS:

These fluids are posses certain degree of elasticity in addition to viscosity. When a viscoelastic fluid is in motion, a certain amount of energy is stored up in the material as strain energy while some energy is lost due to viscous dissipation. In this class of fluids unlike the inelastic viscous fluids, one cannot neglect the strain, however small it may be, as it is responsible for the recovery to the original state and for the possible reverse flow that follows the removal of the stress. During the flow the natural state of fluid changes constantly and tries to attain the instantaneous state of the deformed state, but it does never succeed completely. This lag is a measure of the elasticity or the so called "memory" of the fluids. We now discuss the constitutive equations of various viscoelastic fluids.

1. Oldroyd fluid: The constitutive equation for the above fluid has been proposed by Oldroyd (1950)

$$S_{ik} = \tau_{ik} - Pg_{ij}$$

$$\tau^{ik} + \lambda_1 \frac{D\tau^{ik}}{Dt} + \mu_0 E^{ik} \tau_j^j + v_1 E^{jl} \tau_{jl} g^{ik} = 2\eta_0 \left[E^{ik} + \lambda_2 \frac{DE^{ik}}{Dt} + v_2 E^{jl} E_{jl} g^{ik} \right]$$

and $E_{ik} = \frac{1}{2}\left(U_{k,l} + U_{l,k}\right)$

Where S_{ik} is the stress tensor, U_l denotes the velocity vector, g_{ik} is the metric tensor, P is an isotropic pressure, η_0 is a constant having the dimensions of viscosity, and λ_1, λ_2 (relaxation time and retardation time parameters respectively), μ_0, v_1 *and* v_2 are constants having the dimensions time. The derivative $\frac{D(\,)}{Dt}$ denoted by is the convected derivative of any tensor B^{ik}, defined in the form

$$\frac{DB^{ik}}{Dt} = \frac{\partial B^{ik}}{\partial t} + U^j B^{ik}{}_{,j} + W_m^l B^{mk} + W_m^k B^{lm} - E_m^l B^{mk} - E_m^k B^{im}$$

Where $W_{ik} = \frac{1}{2}\left(U_{k,i} - U_{l,k}\right)$

In this class of fluids all the non-Newtonian flow properties observed in vosco-elastic fluids are present. These properties are present for all rates of shear, when six constants in second equation satisfy the following conditions:

$$\sigma_1 > \sigma_2 \geq \frac{1}{9}$$

Where
$$\sigma_1 = \lambda_1 \mu_0 + (\lambda_1 - 1.5\mu_0)v_1,$$
$$\sigma_2 = \lambda_2 \mu_0 + (\lambda_1 - 1.5\mu_0)v_2,$$

and $\quad (\lambda_1 - 1.5\mu_0)(\lambda_1 v_2 - \lambda_2 v_1) \geq 0.$

2. Rivlin–Ericksen fluids: The general theory put forward so far from purely phenomenological consideration is that by Rivlin-Ericksen. The constitutive equation in this case is

$$S = -pI + \phi_1 A_1 + \phi_2 A_2 + \phi_3 A_1^2 + \phi_4 A_2^2 + \phi_5 (A_1 A_2 + A_2 A_1) + \phi_6 (A_1^2 A_2 + A_2 A_1^2)$$
$$\phi_7 (A_1 A_2^2 + A_2^2 A_1) + \phi_8 (A_1^2 A_2^2 + A_2^2 A_1^2)$$

Where p is an arbitrary hydrostatic pressure and $\phi's$ polynomial functions of the traces of the various tensors occurring in the representation, matrices A_1 and A_2 are defined by

$$A_{ij}^{(1)} = \left(v_{i,j} + v_{j,i}\right)$$
$$A_{ij}^{(2)} = \frac{\partial A_{ij}^{(1)}}{\partial t} + v_p A_{ij,p}^{(1)} + A_{ip}^{(1)} v_{p,j} + A_{pj}^{(1)} v_{p,i}$$

v_p being velocity vector

On neglecting the squares and products of A_2, we have

$$S = -pI + \phi_1 A_1 + \phi_2 A_2 + \phi_3 A_1^2$$

Where ϕ_1, ϕ_2 *and* ϕ_3 are constants. It is customary to call ϕ_1 the coefficient of ordinary viscosity, ϕ_2 the coefficient of viscoelasticity, and ϕ_3 the coefficient of cross-viscosity.

Coleman and Noll (1960) have adopted a different approach to obtain the constitutive equation. In this case, the constitutive equation is

$$S = -pI + \phi_1 E^{(1)} + \phi_2 E^{(2)} + \phi_3 E^{(1)^2}$$

Where
$$E_{ij}^{(1)} = v_{i,j} + v_{j,i}$$
$$E_{ij}^{(2)} = A_{i,j} + A_{j,i} + 2v_{m,i}\, v_{m,j}$$

In the above equations, S is the stress-tensor, v_i and A_i are the components of velocity and acceleration in the direction of the i^{th} coordinates x_i, p is an indeterminate hydrostatic pressure and the coefficients ϕ_1, ϕ_2 *and* ϕ_3 have already been defined above.

It has been reported that solutions of poly-isobutylene in cetane at 30^0C simulate a second order fluid, and the material constants for the solutions of various concentrations have been determined by Markovitz (1964).

3. Walters fluid: The constitutive equation for the Walters liquid B' is given by

$$S_{ik} = -pg_{ik} + \tau_{ik}$$
$$\tau_{ik}(x,t) = 2\int_{-\infty}^{t} \psi(t-t')\frac{dx^i}{dx'^m}\frac{dx^k}{dx'^r}\, e^{(1)mr}(x',t')dt'$$

where S_{ik} is the stress tensor, p an arbitrary isotropic pressure, g_{ik} the metric tensor of a fixed coordinates system, x', x'^i the position at the time t' of the element which is instantaneously at the point x' at time t, $e_{ik}^{(1)}$ the rate of strain tensor.

4. Maxwell fluids: The constitutive equation for this class of fluid has been proposed by Maxwell as $\left(1+\lambda\frac{\partial}{\partial t}\right)\cdot \tau^{ik} = 2\mu e^{ik}$

Where λ is the relaxation time for the stress.

POLAR FLUIDS:

The constitutive equations of three different models characterizing polar fluids are as follows:

1. The model of Condiff and Dahler:

$$\tau_{(ij)} = \left(-p + \frac{\phi - 2\eta}{3} d_{kk}\right) \delta_{ij} + 2\eta d_{ij}$$

$$\tau_{(ij)} = \xi\, \varepsilon_{ijk} \left(\varepsilon_{ksq}\ v_{q,s} - 2_{vk}\right)$$

$$M_{ij} = \left(-\xi_1 + \frac{2\xi_2}{3}\right) v_{k,k}\, \delta_{i,j} + \xi_2 \left(v_{i,j} + v_{j,i}\right)$$

2. The micropolar model of Eringen:

$$\tau_{ij} = (-p + \lambda d_{kk})\delta_{ij} + (2\mu + k) d_{ij} + k_{\varepsilon jk}(w_k - v_k)$$

$$M_{ij} = \alpha v_{p,p}\, \delta_{ij} + \beta v_{i,j} + \gamma v_{j,i}$$

3. The model of Stokes:

$$\tau_{(ij)} = (-p + \lambda d_{kk})\delta_{ij} + 2\mu\, d_{ij}$$

$$\tau_{[ij]} = -2\eta W_{ij,kk} - \frac{p}{2} E_{ij} + G_s$$

$$M_{ij} = 4\eta_{wj,i} + 4\eta' w_{i,j}$$

Where $\tau_{(ij)}$ and $\tau_{[ij]}$ are the symmetric and antisymmetric parts of the stress tensor τ_{ij} and M_{ij} is the couple stress tensor, G is the body couple, ε_{ijk} is the alternating tensor, w is the vorticity and v represents micro rotation.

The parameters $\phi, \eta, \xi, \xi_1, \xi_2, \lambda, \mu, k$... are all material constants being characteristics of each polar fluid model.

DIPOLAR FLUIDS:

The constitutive equations for an incompressible dipolar fluid under isothermal conditions have been proposed by Blenstein and Green

$$\tau_j + \phi\, \delta_{ij} = 2\mu\, d_{ij}$$

$$\Sigma_{(ij)k} + \psi_i\, \delta_{jk} + \psi_j\, \delta_{ik} = h_1\, \delta_{ij} v_{k,m} + h_2 \left(v_{i,jk} + v_{j,ik}\right) + h_3\, v_{k,ji}$$

Where v_i is the velocity components, d_{ij} are the components of the rate of deformation tensor, τ_{ij} are the components of the stress tensor and δ_j is the Kronecker delta. $\Sigma_{(ij)k}$ are the components of dipolar stresses which are symmetric in the first two indices (the antisymmetric part $\Sigma_{(ij)k}$ of $\Sigma_{i,jk}$ in the first two indices does not contribute to the momentum equations), repeated index denotes summation, is the coefficient of viscosity, h_1, h_2 and h_3 are the material constants and are restricted by the inequalities.

$$2h_1 + h_3 \geq 0, \qquad 2h_2 + h_3 \geq 0,$$
$$h_3 - h_2 \geq 0, \qquad 5h_1 + 2h_3 - h_2 \geq 0$$

The functions ϕ *and* ψ_i are related to the hydrostatic pressure p by

$$p = \phi - 2\psi_{i,i}$$

The non-symmetric monopolar stress tensor σ_{ij} and the stress tensor t_{ij} are connected by the relation

$$t_{ij} = \sigma i_j + \Sigma k_{ij,k} - p\left[d^2\left\{\left(\frac{Dv_j}{Dt}\right), i - v_{j,k}\ v_{k,i}\right\} - F_{ij}\right] = \tau_{ji}$$

where p is the density of the fluid , F_{ij} are the dipolar body forces per unit mass, d is a material constant and D/Dt denotes the material derivative.

ANISOTROPIC FLUIDS:

The constitutive equations for incompressible anisotropic fluids in Cartesian form are

$$t_{ij} = -p\,\delta_{ij} + 2\mu d_{ij} + (\mu_1 + \mu_2\, d_{km}\, n_k\, n_m) n_i\, n_j + 2\mu_3\left(d_{jk}\, n_k\, n_i + d_{jk}\, n_k\, n_j\right)$$

With $n_i = w_{ij}\, n_j + \lambda\left(d_{ij}\, n_j - d_{km}\, n_k\, n_m\, n_i\right)$

Where n_i are the vectors indicating preferred directions. It is assumed [consistent with] that

$$n_k\, n_k = 1,$$

this being a simplifying assumption which excludes effects like those of elasticity of suspended particles. Here λ*'s and* μ*'s* are material constants, δ_{ij} is the Kronecker delta, p is the arbitrary isotropic pressure and the dot denotes the material derivative. Here d and w represent the symmetric and antisymmetric parts of the velocity gradient tensor,

$$2d_{ik} = x_{i,k} + x_{k,i}$$

$$2w_{ik} = x_{i,k} - x_{k,i}$$

CONCLUSION:

There are several types of non–Newtonian fluid models of which are proposed by scientists working in this area. Several empirical models are generally used to approximate the experimental data are available for such fluids. Calculations on non-Newtonian media present a new challenge in flow analysis. Simulating these types of flows in order to calculate pipe and pump sizes presents a significant challenge to the engineer. We hope that the stress-strain relationship along with possible proposed generalization for different type of non-Newtonian fluids would be helpful to researchers and engineers for further experimental and theoretical study.

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