ISSN : 2320-2971 (Online)

International e-Journal for Education and Mathematics

www.iejem.org Vol. 02, No. 04, (Aug. 2013), pp 37-43

On the Heat Transfer of Gas Laminar Forced-Convection Flow with variable physical property

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Article Info.

Received on 12 June 2013
Revised on 30 June. 2013
Accepted on 30 June. 2013

Keywords:
heat transfer, forced convection non-Newtonian fluids, Stress-Strain relationship, dimensional analysis

A B S T R A C T

The importance of the study of heat transfer of forced convection in every field of Engineering practice and human life has motivated scientist to investigate such phenomenon rigorously. The Laminar forced convection flow with variable physical properties is discussed. Such flow equations are governed by coupled non-linear boundary layer equations. The similarity variables are systematically derived using well-known dimensional analysis method with all possible restrictions and conditions. It is observed that velocity and Temperature field as well as heat transfer of gas laminar force convection is significantly influenced by the Prandtl no. and the gas temperature parameters together with boundary temperature ratio. The present analysis clarifies that (1) the effect of gas variable physical properties on heat transfer coefficient is dominated by the gas temperature parameter and the boundary temperature ratio. (2) The effect of variable thermal conductivity on heat transfer coefficient is larger than that of the variable absolute viscosity without ignoring the effect of viscosity. (3) Increasing the boundary temperature ratio causes increases of effect of the variable physical properties.

AMS Subject classification: *76A05, 76M55, 54H15*

1. Introduction

Heat transfer of forced convection exists widely in every field of engineering practice and human life, so it is important to rigorously investigate this phenomenon. On the early time, the single Blasius Ordinary Differential Equation transformed from the monument partial differential equations was applied for prediction of its velocity field [1]. Polhausen [2] used Blasius transformation system to further calculate heat transfer with constant property assumption for Laminar forced convection on a horizontal flat plate. After their studies, the investigations have been done for the effect of the variable thermo physical properties on compressible forced boundary layer (Schiliting [3]). For incompressible forced boundary layer there have been some studies with consideration of variable liquid viscosity, for instance, the studies of Ling and Dybbs [4,5],who proposed an assumed equation for approximate simulation of the viscosity of the fluid, where the viscosity of the fluid is an inverse linear function of temperature. However, there still has been a requirement for an extensive investigation of coupled effect of the variable physical properties, such as density, thermal conductivity, and viscosity on heat transfer of forced convection, In recent paper the coupled effect of these variable physical properties on heat transfer with free convection were systematically reported. These studies demonstrated that the variable physical properties have obvious effect on free convention which can never be ignored for reliable prediction of the related heat transfer.

3. Basic Conservation Equations

Considering two-dimensional conservation equations for non Newtonian laminar forced convection boundary layer are shown below.

$$\frac{\partial w_x}{\partial x}+\frac{\partial w_y}{\partial y}=0 \tag{1}$$

$$w_x\frac{\partial w_x}{\partial x}+w_y\frac{\partial w_x}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial}{\partial y}\left[\left|\frac{\partial w_x}{\partial y}\right|^n\right] \tag{2}$$

$$w_x\frac{\partial t}{\partial x}+w_y\frac{\partial t}{\partial y}=\frac{\nu}{P_r}\frac{\partial^2 t}{\partial y^2} \tag{3}$$

With boundary conditions

$$y=0: w_x=0, w_y=0, t=t_w \tag{4}$$

$$y\to\infty: w_x=w_{x,\infty}=(\text{constant}), t=t_\infty$$

Introducing the stream function ψ

$$w_x=\frac{\partial\psi}{\partial y}, w_y=-\frac{\partial\psi}{\partial x} \tag{5}$$

In Equation (1), (2) and (3) we get,

$$\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial^2\psi}{\partial x\partial y}=0 \quad \text{(gets satisfied identically)}$$

$$\frac{\partial\psi}{\partial y}.\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}.\frac{\partial^2\psi}{\partial y^2}=w_{x,\infty}\frac{dw_{x,\infty}}{dx}+\nu n\left|\frac{\partial^2\psi}{\partial y^2}\right|^{n-1}\frac{\partial^3\psi}{\partial y^3} \tag{6}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial t}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial t}{\partial y}=\frac{\nu}{p_r}\frac{\partial^2 t}{\partial y^2} \tag{7}$$

Where, $-\frac{1}{\rho}\frac{\partial p}{\partial x}=w_{x,\infty}\frac{dw_{x,\infty}}{dx}$

Subject to the boundary conditions:

$$y=0:\frac{\partial\psi}{\partial y}=0,\frac{\partial\psi}{\partial x}=0,t=t_w$$
$$y\to\infty:\frac{\partial\psi}{\partial y}=w_{x,\infty}=(\text{constant}),t=t_\infty \tag{8}$$

Equations (6)-(8) are nonlinear partial differential equations containing two independent variables and hence to reduce it into ordinary differential equations we select followinf one-parameter group of transformation:

$$x=A^{\alpha_1}\overline{x},\quad y=A^{\alpha_2}\overline{y},\quad \psi=A^{\alpha_3}\overline{\psi},\quad w_{x,\infty}=A^{\alpha_4}\overline{w_{x,\infty}} \tag{9}$$

Introducing Transformations (9) into equations (6)-(8) and usingsimple chain rule, and then eploring condition of invariance we get folowing relations among α 's

$$\frac{\alpha_2}{\alpha_1}=a\ (say)\quad \text{and}\quad \frac{\alpha_3}{\alpha_1}=-\frac{(1-2n)a+1}{2-n}$$

Now in second step, we can easily derive following similarity transformations;

$$\eta=\overline{y}.\overline{x}^{s}\quad \text{and } f(\eta)=\overline{\psi}.\overline{x}^{r} \tag{10}$$

Where 's' and 't' are given by:

$$s=-\frac{\alpha_2}{\alpha_1}=-a\ (say)\quad \text{and } t=-\frac{\alpha_3}{\alpha_1}=-\frac{(1-2n)a+1}{2-n} \tag{11}$$

Thus, the new similarity transformations are

$$\eta=yx^{-a}\quad \text{and } f(\eta)=\frac{\psi}{x^{\left[\frac{(1-2n)a+1}{2-n}\right]}} \tag{12}$$

Again from the boundary conditions we have,

$$w_x=w_{x,\infty}$$

But $w_x=\dfrac{\partial\psi}{\partial y}=\dfrac{\partial}{\partial y}\left(x^{\alpha}f(\eta)\right)=x^{\alpha-a}f'(\eta)$ where $\alpha=\dfrac{(1-2n)a+1}{2-n}$

$$\Rightarrow w_{x,\infty}=x^{\alpha-a}f'(\eta)$$

$$\Rightarrow\ w_{x,\infty}\ \alpha\ x^{\alpha-a}\quad \Rightarrow w_{x,\infty}=Cx^{\alpha-a}\quad \text{where C is constant} \tag{13}$$

$$\frac{\partial\psi}{\partial y}=\frac{\partial}{\partial y}\left(x^{\alpha}f(\eta)\right)=x^{\alpha-a}f'(\eta)$$

$$\frac{\partial\psi}{\partial x}=\frac{\partial}{\partial x}\left(x^{\alpha}f(\eta)\right)=x^{\alpha-1}[\alpha f-a\eta f'(\eta)]$$

$$\frac{\partial^2 \psi}{\partial x \partial y} = \frac{\partial}{\partial x}\left(x^{\alpha-a} f'(\eta) \right) = x^{\alpha-a-1}[(\alpha\text{-}a)f'(\eta) - a\eta f''(\eta)]$$

$$\frac{\partial^2 \psi}{\partial y^2} = \frac{\partial}{\partial y}\left(x^{\alpha-a} f'(\eta) \right) = x^{\alpha-2a} f''(\eta)$$

$$\frac{\partial^3 \psi}{\partial y^3} = \frac{\partial}{\partial y}\left(x^{\alpha-2a} f''(\eta) \right) = x^{\alpha-3a} f'''(\eta) \quad \text{and}$$

$$\frac{dw_{x,\infty}}{dx} = (\alpha - a)Cx^{\alpha-a-1}$$

Substituting all these values in Equation (6) and (7) we get,

$$(\alpha - a)f'^2(\eta) - \alpha f(\eta)f''(\eta) = -C^2(\alpha\text{-}a) + [\nu n x^{\frac{4a(n-1)}{2-n}}] f''^{(n-1)}(\eta) f'''(\eta) \tag{14}$$

Taking C = 1and $[\nu n x^{\frac{4a(n-1)}{2-n}}]$ =β (Some Non dimensional Quantity)

The Above Equation is reduced to

$$(\alpha - a)f'^2(\eta) - \alpha f(\eta)f''(\eta) = -(\alpha\text{-}a) + \beta f''^{(n-1)}(\eta) f'''(\eta)$$
(15)

And equation (7) will reduce to:

$$x^{1-2a}[-\alpha f(\eta)\theta'(\eta)] = \frac{\nu}{p_r}\theta''(\eta)$$

$p_r f(\eta)\theta'(\eta) = \gamma\theta''(\eta)$ where $\frac{1}{\gamma} = \frac{\alpha x^{1-2a}}{\nu}$ (Non dimensional quantity)

(16) with Boundary conditions:

$$\eta = 0, f'(\eta) = 0, f(\eta) = 0, \theta(\eta) = 1$$
$$\eta = \infty, f'(\eta) = 1, \theta(\eta) = 0$$

For Newtoniancase i.e. for n=1, set of equations (15)-(17) will be reduced to those derived by Shang [7]

4. Governing Partial differential equation with variable Physical property.

A flat plate is horizontally located in parallel main stream velocity $w_{x,\infty}$. The plate surface temperature is t_w and fluid bulk temperature is t_∞. Then a velocity boundary layer will occur near the plate. We assume the velocity boundary layer is laminar.

$$\frac{\partial(\rho w_x)}{\partial x} + \frac{\partial(\rho w_y)}{\partial y} = 0 \tag{17}$$

$$\left(\rho w_x \frac{\partial \rho w_x}{\partial x} + \rho w_y \frac{\partial \rho w_x}{\partial y}\right) = \frac{\partial}{\partial y}\left[\left|\frac{\partial w_x}{\partial y}\right|^n \mu\right] \tag{18}$$

$$\rho w_x \frac{\partial (C_p t)}{\partial x} + \rho w_y \frac{\partial (C_p t)}{\partial y} = \frac{\partial}{\partial y}\left[\lambda \frac{\partial t}{\partial y}\right] \tag{19}$$

With Boundary conditions

$$\begin{aligned} & y = 0 : \rho w_x = 0, \rho w_y = 0, t = t_w \\ & y \to \infty : \rho w_x = \rho w_{x,\infty} = (cons\tan t), t = t_\infty \end{aligned} \tag{20}$$

Here, temperature dependent physical properties density ρ, Absolute viscosity μ, thermalconductivity λ, and specific heat C_p are taken into account.As ρ is a variable phyisical property the above Equations are further simplified as

$$\frac{\partial (\rho w_x)}{\partial x} + \frac{\partial (\rho w_y)}{\partial y} = 0 \tag{21}$$

$$\rho\left(w_x \frac{\partial \rho w_x}{\partial x} + w_y \frac{\partial \rho w_x}{\partial y}\right) = \mu n \left|\frac{\partial \rho w_x}{\partial y}\right|^{n-1} \frac{\partial^2 \rho w_x}{\partial y^2} + \left(\frac{\partial \rho w_x}{\partial y}\right)^n \frac{\partial \mu}{\partial y} \tag{22}$$

$$\rho\left(w_x C_p \frac{\partial (t)}{\partial x} + w_x t \frac{\partial (C_p)}{\partial x} + w_y C_p \frac{\partial (t)}{\partial y} + w_y t \frac{\partial (C_p)}{\partial y}\right) = \lambda \frac{\partial^2 t}{\partial y^2} + \frac{\partial \lambda}{\partial y}\frac{\partial t}{\partial y} \tag{23}$$

With same boundary conditions

Now solving the boundary condition $t = t_w$ and $t = t_\infty$ we get,

$$\theta(\eta) = \frac{t - t_\infty}{t_w - t_\infty} \tag{24}$$

Introducing the stream function ψ

$$\rho w_x = \frac{\partial \psi}{\partial y}, \qquad \rho w_y = -\frac{\partial \psi}{\partial x}$$

Hence equation (21)-(23) will be

$$\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial x \partial y} = 0 \quad \text{(Identically satisfied)}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial y^2} = \mu n \left|\frac{\partial^2 \psi}{\partial y^2}\right|^{n-1} \frac{\partial^3 \psi}{\partial y^3} + \left(\frac{\partial^2 \psi}{\partial y^2}\right)^n \frac{\partial \mu}{\partial y}$$

$$\frac{\partial \psi}{\partial y} C_p \frac{\partial (t)}{\partial x} + \frac{\partial \psi}{\partial y} t \frac{\partial (C_p)}{\partial x} - \frac{\partial \psi}{\partial x} C_p \frac{\partial (t)}{\partial y} - \frac{\partial \psi}{\partial x} t \frac{\partial (C_p)}{\partial y} = \lambda \frac{\partial^2 t}{\partial y^2} + \frac{\partial \lambda}{\partial y}\frac{\partial t}{\partial y}$$

Using similarity transformations defined in Equation (12) and (24), the above last two equations are reduced to an ordinary differential Equations as

$$(\alpha - a) f'^2(\eta) - \alpha f(\eta) f''(\eta) = \mu \, \mathrm{n} f''^{(n-1)}(\eta) f'''(\eta) + \frac{\partial \mu}{\partial \eta} \tag{25}$$

$$x^{\frac{(1-n)(3a-1)}{2-n}} [-\alpha C_p f(\eta)\theta'(\eta) - \alpha f(\eta) t \frac{\partial C_p}{\partial \eta}] = \lambda \theta''(\eta) + \frac{\partial \lambda}{\partial \eta} \tag{26}$$

Boundary conditions

$$\eta = 0, f'(\eta) = 0, f(\eta) = 0, \theta(\eta) = 1$$

$$\eta = \infty, f'(\eta) = 1, \theta(\eta) = 0$$

For Newtoniancase i.e. for n=1, set of equations (25) and(26) will be reduced to those derived by Shang [7].

5. Conclusion:

Equations (26) and (27) are completely ordinary differential equations of laminar forced convection. These transformed governing ordinary differential equations are completely dimensionless because of the followings: (1) They involve dimensionless velocity components f(η), dimensionless temperature θ(η), as well as their dimensionless derivatives, which constitute whole unknown variables of governing equations; (2) All physical properties exist in form of the dimensional physical property factors, such as

$$\frac{1}{\rho}\frac{\partial \rho}{\partial \eta}, \frac{1}{C_p}\frac{dC_p}{d\eta}, \frac{1}{\lambda}\frac{d\lambda}{d\eta}, \frac{t_\infty}{t_w - t_\infty}.$$

The coupled effect of the variable physical properties on laminar forced convection is dominated by these dimensionless physical property factors.

Similarity solutions of two dimensional conservation equation for Non-Newtonian (power law fluids) laminar forced convection boundary layer on a horizontal flat plate is investigated using one parameter Linear group of Transformation.

The similarity equations derived are agreed with that of derived by "Shang" for the case of Newtonian fluid. The similarity transformations with some little change are is used to derive similarity solution for the Partial Differential Equation governing the two-dimensional laminar forced convection flow of Non-Newtonian power law fluids with same flow geometry but with variable physical properties like ρ, μ, $C_{p,}$ λ. The advantage of present technique is it is simple and hence it works well with many complicated highly nonlinear partial differential equation governing the different flow geometry.

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