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# On the Class of Similarity Solution for electrically conducting Non-Newtonian fluids over a Vertical porous- elastic Surface

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**A B S T R A C T**

*Similarity solution is investigated for a free convective boundary layer flow of electrically conducting Non-Newtonian fluids over a vertical porous-elastic surface. The similarity equation is derived using one parameter linear group of transformation. Finally, this similarity equation which is highly non-linear ordinary differential equation is solved numerically for particular Non-Newtonian fluid so-called Powell-Eyring fluid.*

## Introduction

The influence of magnetic field on viscous incompressible flow of electrically conducting fluid has its importance in many applications such as extrusion of plastics in the manufacture of Rayon and Nylon , purification of crude oil, pulp, paper industry, textile industry and in different geophysical cases. Also, the transpiration cooling is considered to be very effective process to protect certain structural elements such as combustion chamber walls, exhaust nozzle walls or gas turbine blades of turbo jets and rocket engines, from the influence of hot gases. In many process industries, the cooling of threads or sheets of some polymer materials is of importance in the production line. The rate of cooling can be controlled effectively to achieve final products of desired characteristics by drawing threads, etc. in the presence of an electrically conducting fluid subject to a magnetic field. Saxena and Singh [1], considered steady two –dimensional MHD heat and mass transfer free convection flow of an incompressible, viscous, electrically conducting and chemically reacting fluid via a porous medium along a continuously moving isothermal vertical surface in the presence of the magnetic field with heat source and transpiration.

Visco-elastic MHD convection flow problem are very important in both theoretical and experimental studies as they have overwhelming implications in various fields as petroleum industries, cooling of nuclear reactors, boundary layer control in aerodynamics, crystal growth etc. A few representative areas of interest in which heat and mass transfer combined along with the chemical reaction play significant role in chemical industries like in food processing and polymer production. Gupta, Pop and Soundalgekar [2], analyzed free convection flow past a linearly accelerated vertical plate in the presence of energy dissipation. Saxena and Dubey [3] have analyzed the flow behavior of unsteady MHD heat and mass transfer free convection flow of a polar fluid past a vertical moving porous plate in a porous medium with heat generation and thermal diffusion. Mass transfer effects on MHD viscous flow past an impulsively started infinite vertical plate with constant mass flux has been studied by Saravana, Sreekant and Sreenadh [4]. The two dimensional free convective hydro-magnetic flow of an electrically conducting visco-elastic fluid past a vertical porous surface has been studied in presence of radiation and chemical reaction of first order by Choudhury and Dhar [5]. Alharbi, Bazid and Gendy [6] studied heat and mass transfer in MHD visco-elastic fluid flow through a porous medium over a stretching sheet with chemical reaction.

Darji and Timol [7] presented an interesting result on deductive group theoretic analysis for MHD flow of a Sisko fluid in a porous medium. Mahdy [8] considered the effect of chemical reaction and heat generation or absorption on double-diffusive convection from a vertical truncated cone in a porous media with variable viscosity. Soundalgekar [9] analyzed the viscous dissipation effects on unsteady free convective flow past an infinite vertical porous plate with constant suction. Sahel, Mohamed and Mohmoud [10] considered the heat and mass transfer in MHD visco-elastic fluid flow through a porous medium over a stretching with chemical reaction.

Darji and Timol [11] analyzed the free convective boundary-layer problem due to the motion of an elastic surface into an electrically conducting a class of non-Newtonian fluid. So motivated by this, we produce similarity solution for a free convective boundary layer flow of electrically conducting Non-Newtonian fluids over a vertical porous and elastic surface. The class of all non-Newtonian fluids is characterized by the property that its stress tensor component $\tau_{ij}$ can be related to the strain rate component $e_{ij}$ by the arbitrary continuous functional relation

$$\mathcal{F}(\tau_{ij}, e_{ij}) = 0 \qquad (1)$$

**Problem Formulation**

We consider a free convective, laminar boundary layer flow of an electrically conducting incompressible viscous power law fluid over a vertical porous and elastic surface. The surface is stretched vertically upward along the positive $x$-axis, with a prescribed velocity

$$y = 0, \qquad u(x, 0) = u_0(x) \qquad (2)$$

While the origin $(x, y) = (0,0)$ is kept fixed. The y-axis is vertical to the surface. Also, due to the fact that the elastic surface is porous, there is a component of the velocity of the fluid which has vertical direction to the surface given by

$$y = 0, \qquad v(x, 0) = v_0(x) \qquad (3)$$

The motion of the surface within the fluid creates a boundary layer, which is extended along the x-axis. The whole system is under the influence of a magnetic field B(x) which applies to the y-direction. We consider that the temperature of the surface changes along the x-axis and its distribution is described by a given function $T_0(x)$. The stress-strain relation, under the boundary layer assumption can be found in the form of arbitrary function with only non-vanishing component. Then equation (1) can be given by

$$\mathcal{F}\left(\tau_{yx}, \frac{\partial u}{\partial y}\right) = 0 \qquad (4)$$

Under the assumption that the viscous dissipation term in the energy equation and the induced magnetic field can be neglected, the basic boundary layer equations of the momentum and energy for the steady flow of Boussinesq type are respectively as follows, with the stress-strain relationship given by (4)

**Momentum**

$$\mathbf{u}\frac{\partial \mathbf{u}}{\partial \mathbf{x}}+\mathbf{v}\frac{\partial \mathbf{u}}{\partial \mathbf{y}}=\frac{\mathbf{1}}{\boldsymbol{\rho}}\frac{\partial}{\partial \mathbf{y}}(\boldsymbol{\tau}_{\mathbf{yx}})-\frac{\boldsymbol{\sigma}\mathbf{B}^2}{\boldsymbol{\rho}}\mathbf{u}+\ \mathbf{g}\boldsymbol{\beta}(\mathbf{T}-\mathbf{T}_\infty) \tag{5}$$

**Energy**

$$\mathbf{u}\frac{\partial \mathbf{T}}{\partial \mathbf{x}}+\mathbf{v}\frac{\partial \mathbf{T}}{\partial \mathbf{y}}=\boldsymbol{\alpha}\frac{\partial^2 \mathbf{T}}{\partial \mathbf{y}^2} \tag{6}$$

Where σ-electric conductivity, β-volumetric coefficient of thermal expansion, ρ-mass density and α-thermal diffusivity, which are assumed to be constants. Also, g-gravity field assumed to be parallel to the x-axis, $T = T(y, x)$ - temperature field and $T_\infty$- temperature at infinity. Therefore the boundary conditions of the problem of the form

$$\mathbf{y}=\mathbf{0},\qquad \mathbf{u}(\mathbf{x},\mathbf{0})=\mathbf{u}_\mathbf{0}\ ,\qquad \mathbf{v}(\mathbf{x},\mathbf{0})=\mathbf{v}_\mathbf{0},\qquad \boldsymbol{\theta}=\boldsymbol{\theta}_\mathbf{0} \tag{7}$$

$$\mathbf{y}=\infty,\qquad \mathbf{u}(\mathbf{x},\mathbf{y})=\mathbf{0}\ ,\quad \boldsymbol{\theta}=\mathbf{0} \tag{8}$$

Where $\theta = T - T_\infty$ , $\theta_0 = T_0 - T_\infty$ is a prescribed function along the boundary surface $y = 0$

**With the stress-strain relationship**

$$\boldsymbol{\mathcal{F}}\left(\boldsymbol{\tau}_{\mathbf{yx}},\frac{\partial \mathbf{u}}{\partial \mathbf{y}}\right)=\mathbf{0} \tag{9}$$

**The above equation can be made dimenstionless using following quantities,**

$$\mathbf{x}^*=\frac{\mathbf{G_r}}{\mathbf{L}}\mathbf{x}\ ,\mathbf{y}^*=\frac{\mathbf{y}}{\mathbf{L}}(\mathbf{R}_{\mathbf{e_x}}.\mathbf{G}_{\mathbf{r_x}})^{\frac{1}{2}}\ ,\mathbf{u}^*=\frac{\mathbf{u}}{\mathbf{U_0}}\ ,\mathbf{R}_{\mathbf{x_e}}=\frac{\mathbf{U_0L}}{\mathbf{v}}$$

$$\mathbf{v}^*=\frac{\mathbf{v}}{\mathbf{U_0}}(\frac{\mathbf{R}_{\mathbf{e_x}}}{\mathbf{G}_{\mathbf{r_x}}})^{\frac{1}{2}}\ ,\boldsymbol{\tau}^*_{\mathbf{yx}}=\frac{\boldsymbol{\tau}_{\mathbf{yx}}}{\boldsymbol{\rho}\mathbf{U}_\mathbf{0}^2}(\frac{\mathbf{R}_{\mathbf{e_x}}}{\mathbf{G}_{\mathbf{r_x}}})^{-\frac{1}{2}}\ ,\boldsymbol{\theta}^*=\frac{\boldsymbol{\theta}}{(\mathbf{T_0}-\mathbf{T}_\infty)}$$

$$\boldsymbol{\theta}^*_\mathbf{0}=\frac{\boldsymbol{\theta}}{(\mathbf{T_0}-\mathbf{T}_\infty)}\ ,\mathbf{P_r}=\frac{\mathbf{v}}{\boldsymbol{\alpha}}\ ,\mathbf{G_r}=\frac{\mathbf{L}^3}{\mathbf{v}^2}\mathbf{g}\boldsymbol{\beta}(\mathbf{T_0}-\mathbf{T}_\infty) \tag{10}$$

**Introducing above non-dimensional quantities in equations (5)-(6),**

**We get**

**Momentum**

$$\mathbf{u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{\partial}{\partial y^*}\left(\tau^*{}_{y^*x^*}\right) - M^* u^* + \lambda\theta^*} \tag{11}$$

**Energy**

$$\mathbf{u^* \frac{\partial \theta^*}{\partial x^*} + v^* \frac{\partial \theta^*}{\partial y^*} = \frac{1}{P_r}\frac{\partial^2 \theta^*}{\partial y^{*2}}} \tag{12}$$

**With the stress-strain relationship**

$$\mathcal{F}\left(\tau^*{}_{yx}, \frac{\partial u^*}{\partial y^*}\right) = 0 \tag{13}$$

**With the boundary conditions**

$$\mathbf{y = 0, \qquad u^*(x,0) = u_0^*, \qquad v^*(x,0) = v_0^*, \qquad \theta^* = \theta_0^*} \tag{14}$$

$$\mathbf{y = \infty, \qquad u^*(x,y) = 0\ , \qquad \theta^* = 0} \tag{15}$$

**Introducing stream function ψ such that,**

$$\mathbf{u^* = \frac{\partial \psi^*}{\partial y^*} \qquad , v^* = -\frac{\partial \psi^*}{\partial x^*}} \tag{16}$$

**Equations (11)-(15) become**

$$\mathbf{\frac{\partial \psi^*}{\partial y^*}\frac{\partial^2 \psi^*}{\partial x^* \partial y^*} - \frac{\partial \psi^*}{\partial x^*}\frac{\partial^2 \psi^*}{\partial y^{*2}} = \frac{\partial}{\partial y^*}\left(\tau_{yx}^*\right) - M^* \frac{\partial \psi^*}{\partial y^*} + \lambda\theta^*} \tag{17}$$

$$\mathbf{\frac{\partial \psi^*}{\partial y^*}\frac{\partial \theta^*}{\partial x^*} - \frac{\partial \psi^*}{\partial x^*}\frac{\partial \theta^*}{\partial y^*} = \frac{1}{P_r}\frac{\partial^2 \theta^*}{\partial y^{*2}}} \tag{18}$$

$$\mathcal{F}\left(\tau^*{}_{yx}, \frac{\partial^2 \psi^*}{\partial y^{*2}}\right) = 0 \tag{19}$$

**With boundary conditions**

$$y = 0, \qquad \frac{\partial \psi^*}{\partial y^*} = u_0 , \qquad \frac{\partial \psi^*}{\partial x^*} = v_0, \qquad \theta^* = \theta_0^* \tag{20}$$

$$y = \infty, \qquad \frac{\partial \psi^*}{\partial y^*} = 0 , \qquad \theta^* = 0 \tag{21}$$

**By using linear group transformation**

$$\bar{x}^* = D^{\alpha_1} x^* , \qquad \bar{y}^* = D^{\alpha_2} y^*, \qquad \bar{\psi}^* = D^{\alpha_3} \psi^*$$

$$\bar{\theta}^* = D^{\alpha_4} \theta^* , \qquad \bar{\tau}^*_{yx} = D^{\alpha_5} \tau^*_{yx} , \qquad \bar{M}^* = D^{\alpha_6} M^* \tag{22}$$

Where $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$ and P are constants

**For the dependent and independent variables. From Eq. (22) one obtains**

$$\left(\frac{\bar{x}^*}{x^*}\right)^{\frac{1}{\alpha_1}} = \left(\frac{\bar{y}^*}{y^*}\right)^{\frac{1}{\alpha_2}} = \left(\frac{\bar{\psi}^*}{\psi^*}\right)^{\frac{1}{\alpha_3}} = \left(\frac{\bar{\theta}^*}{\theta^*}\right)^{\frac{1}{\alpha_4}} = \left(\frac{\bar{\tau}^*_{yx}}{\tau^*_{yx}}\right)^{\frac{1}{\alpha_5}} = \left(\frac{\bar{M}^*}{M^*}\right)^{\frac{1}{\alpha_6}} = D \tag{23}$$

**Introducing the linear transformation, given by equation (23), into the equations (17)-(19) result in**

$$D^{\alpha_1+2\alpha_2-2\alpha_3} \frac{\partial \bar{\psi}^*}{\partial \bar{y}^*} \frac{\partial^2 \bar{\psi}^*}{\partial \bar{x}^* \partial \bar{y}^*} - D^{\alpha_1+2\alpha_2-2\alpha_3} \frac{\partial \bar{\psi}^*}{\partial \bar{x}^*} \frac{\partial^2 \bar{\psi}^*}{\partial \bar{y}^{*2}}$$

$$= D^{\alpha_2-\alpha_5} \frac{\partial}{\partial \bar{y}^*}\left(\bar{\tau}^*_{yx}\right) - D^{\alpha_2-\alpha_3-\alpha_6} \bar{M}^* \frac{\partial \bar{\psi}^*}{\partial \bar{y}^*} + \lambda D^{-\alpha_4} \bar{\theta}^* \tag{24}$$

$$D^{\alpha_1+\alpha_2-\alpha_3-\alpha_4} \frac{\partial \bar{\psi}^*}{\partial \bar{y}^*} \frac{\partial \bar{\theta}^*}{\partial \bar{x}^*} - D^{\alpha_1+\alpha_2-\alpha_3-\alpha_4} \frac{\partial \bar{\psi}^*}{\partial \bar{x}^*} \frac{\partial \bar{\theta}^*}{\partial \bar{y}^*} = \frac{1}{P_r} D^{2\alpha_2-\alpha_4} \frac{\partial^2 \bar{\theta}^*}{\partial \bar{y}^{*2}} \tag{25}$$

**And**

$$\mathcal{F}\left(D^{\alpha_5}\, \bar{\tau}_{yx} , \qquad p^{-\alpha_3+2\alpha_2} \frac{\partial^2 \bar{\psi}}{\partial \bar{y}^2}\right) = 0 \tag{26}$$

The differential equation are completely invariant to the proposed linear transformation, the following coupled algebraic equations are obtained

$$\alpha_1 + 2\alpha_2 - 2\alpha_3 = -\alpha_4 \tag{27}$$

$$\alpha_2 - \alpha_5 = -\alpha_4 \tag{28}$$

$$\alpha_2 - \alpha_3 - \alpha_6 = -\alpha_4 \tag{29}$$

$$\alpha_1 + \alpha_2 - \alpha_3 - \alpha_4 = 2\alpha_2 - \alpha_4 \tag{30}$$

$$-\alpha_3 + 2\alpha_2 \quad = 0 \tag{31}$$

**and**

$$\alpha_5 = 0 \tag{32}$$

**By Solving above equations, we get**

$$\alpha_2 = \frac{1}{3}\alpha_1 = \frac{1}{2}\alpha_3 = -\frac{1}{2}\alpha_6 \text{ and } \alpha_5 = 0 \tag{33}$$

**Introducing equation (33) into equation(23) results in**

$$\eta = \frac{y^*}{x^{*\frac{1}{3}}}, \qquad \psi^* = f(\eta)x^{*\frac{2}{3}}, \qquad \theta^* = G(\eta)x^{*-\frac{1}{3}}$$

$$\tau_{yx}^* = H(\eta), \qquad M^* = x^{*-\frac{2}{3}}\, m \tag{34}$$

**With boundary conditions, equations (20)-(21) becomes**

$$\eta = 0, \qquad f'(0) = u_0, f(0) = v_0, G(0) = \theta_0$$

$$\eta \to \infty, \qquad f'(\infty) = 0, \qquad G(\infty) = 0 \tag{35}$$

**Introducing equation (34) in equations (17)-(21), we get following similarity equation**

$$f'^2(\eta) - 2f(\eta)f''(\eta) - 3H'(\eta) + 3m.f'(\eta) - 3\lambda G(\eta) = 0 \tag{36}$$

$$2f(\eta)G'(\eta) + f'(\eta)G(\eta) + \frac{1}{P_r}G''(\eta) = 0 \tag{37}$$

$$\mathcal{F}(H(\eta), f''(\eta)) = 0 \tag{38}$$

**With boundary conditions**

$$\eta = 0, \qquad f(0) = c_1, \qquad f'(0) = c_2, \qquad G(0) = c_3$$

$$\eta \to \infty, \qquad f'(\infty) = 0, \qquad G(\infty) = 0 \tag{39}$$

**Result and Discussions**

Many Non-Newtonian fluid models based on functional relationship between shear-stress and rate of the strain, are available in real world applications Bird [12]. Among these models most research work is so far carried out on power-law fluid model, this is because of its mathematical simplicity. On the other hand rest of fluid models is mathematically more complex and the natures of partial differential equations governing these flows are too non-linear boundary value type and hence their analytical or numerical solution is bit difficult. For the present study the partial differential equation model, although mathematically more complex, is chosen mainly due to two reasons. Firstly, it can be deduced from kinetic theory of liquids rather than the empirical relation as in power-law model. Secondly, it correctly reduces to Newtonian behavior for both low and high shear rate. This reason is somewhat opposite to Pseudo plastic system whereas the power-law model has infinite effective viscosity for low shear rate and thus limiting its range of applicability.

**Mathematically, the Powell-Eyring model can be written as (Bird [12], Skelland [13])**

$$\tau_{yx=}\mu\frac{\partial u}{\partial y}+\frac{1}{B}\sin h^{-1}\left(\frac{1}{C}\frac{\partial u}{\partial y}\right) \tag{40}$$

Where B and C are rheological parameters

Introducing the dimensionless quantities into equation (40) and using similarity variables, we get

$$H'(\eta)=f''' + \frac{\epsilon_1 f'''}{\sqrt{1+\epsilon_2 f''^2}} \tag{41}$$

Where $\epsilon_1=\frac{1}{\mu BC}$ , $\epsilon_2=\frac{\rho u_0^3 G_r}{\mu L C^2}$ are referred as rheological flow parameters.

**Substituting the value from (41), the system (36-38) reduce to,**

$$F'''=\frac{\frac{1}{3}\left(f'^2-2ff''+3mf'-\lambda G\right)\sqrt{1+\epsilon_2 f''^2}}{\epsilon_1+\sqrt{1+\epsilon_2 f''^2(\eta)}} \tag{42}$$

$$2f(\eta)G'(\eta)+f'(\eta)G(\eta)+\frac{1}{P_r}G''(\eta)=0 \tag{43}$$

**Also the dimensionless local skin-friction confident $C_{fx}$ expression is given by**

$$\frac{1}{2}C_{fx}\sqrt{Re_x Gr_x}\equiv\tau_w \tag{44}$$

**Where $\tau_w$ is local shear stress. That is $\tau_w=\tau_{yx}|_{y=0}$**

**In terms of defined rheological flow parameters (44) yields,**

$$\frac{1}{2}C_{fx}\sqrt{Re_x Gr_x}=\epsilon_2 f''(0)+\frac{\epsilon_1}{\sqrt{\epsilon_2}}\sin h^{-1}\left(\sqrt{\epsilon_1}f''(0)\right) \tag{45}$$

In order to face numerically problem (36)-(39), we have used a numerical solver of MATLAB package which solves any two-point boundary value problem for ODEs by collocation. To enhance the effect of magnetic field, without loss of generality, each parameter assumed appropriately in boundary conditions (39). The numerical solutions are produced graphically in Figures (1)-(3).

**Figure (1) shows that boundary layer decrease as the magnetic field increase.**

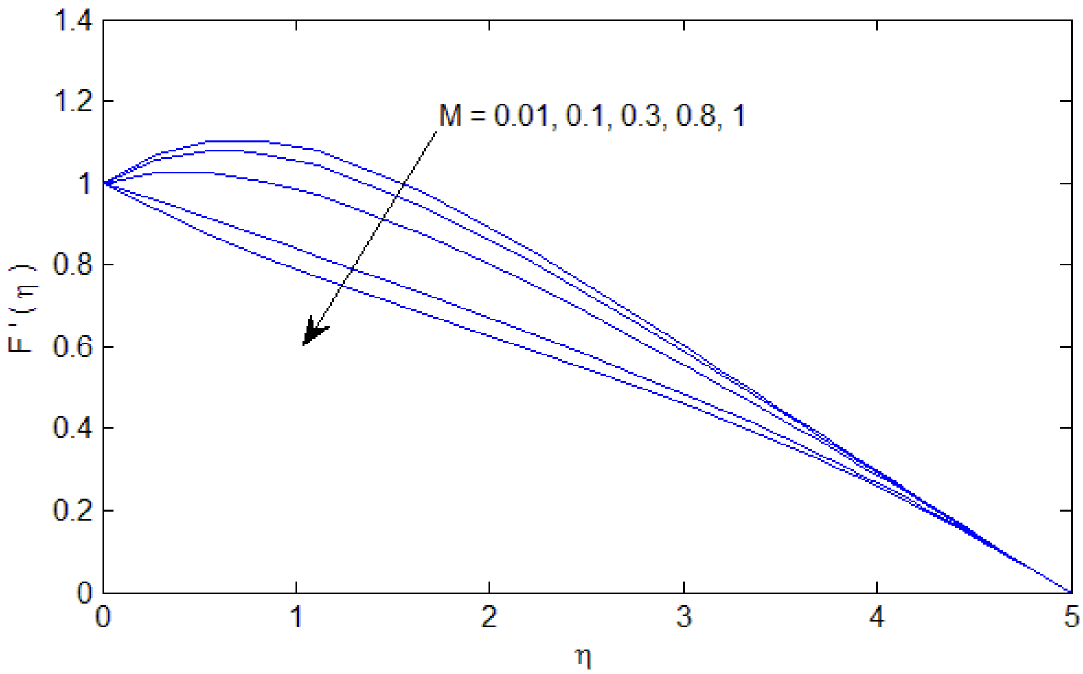


**Figure 1: Influence of magnetic field on horizontal velocity**

Figure (2) depicts behavior of $f''(\eta)$ throughout the domain. In particular it is interesting to observe that, as M increases $f''(0)$ decrease and hence the local shear-stress (see Table 1), which decreases local skin-friction $C_f$.

**Table 1: Local shear stress**

| **M** | **0.01** | **0.1** | **0.3** | **0.8** | **1** |
|---|---|---|---|---|---|
| $f''(0)$ | **0.3404** | **0.2736** | **0.1414** | **-0.1477** | **-0.2518** |
| $\tau_w$ | **0.6717** | **0.5397** | **0.2816** | **-0.2949** | **-0.4868** |

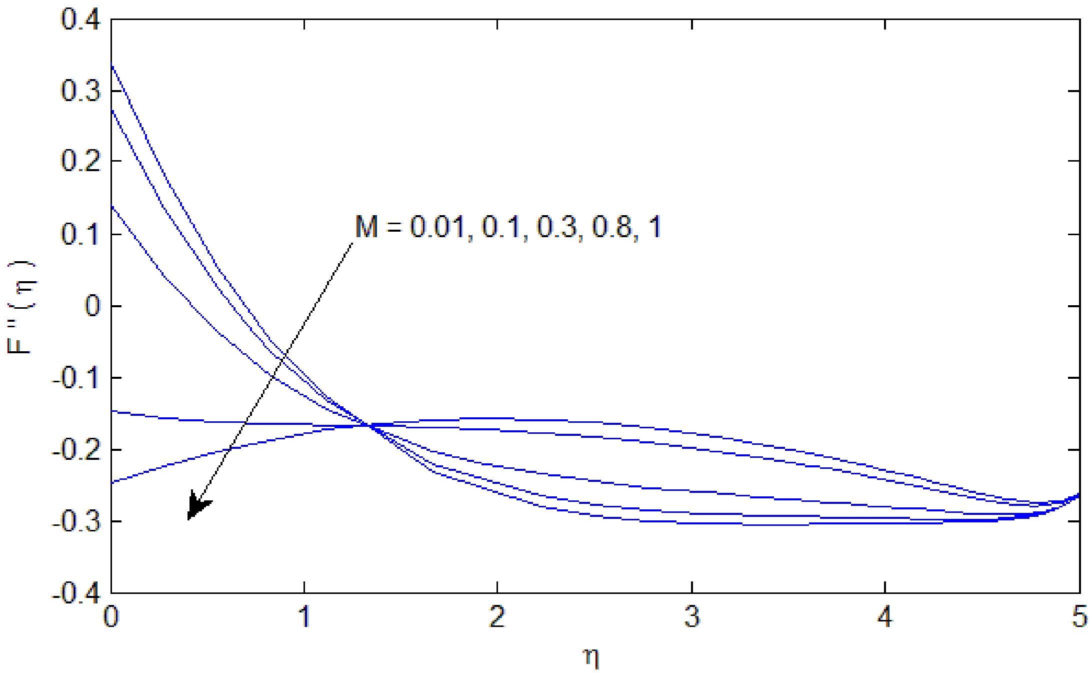


**Figure 2: Influence of magnetic field on shear stress within boundary layer domain**

Influence of magnetic field on thermal boundary layers displayed by Figure 3. It shows that increase in magnetic field will precisely increase thermal boundary layer within the boundary layer domain.

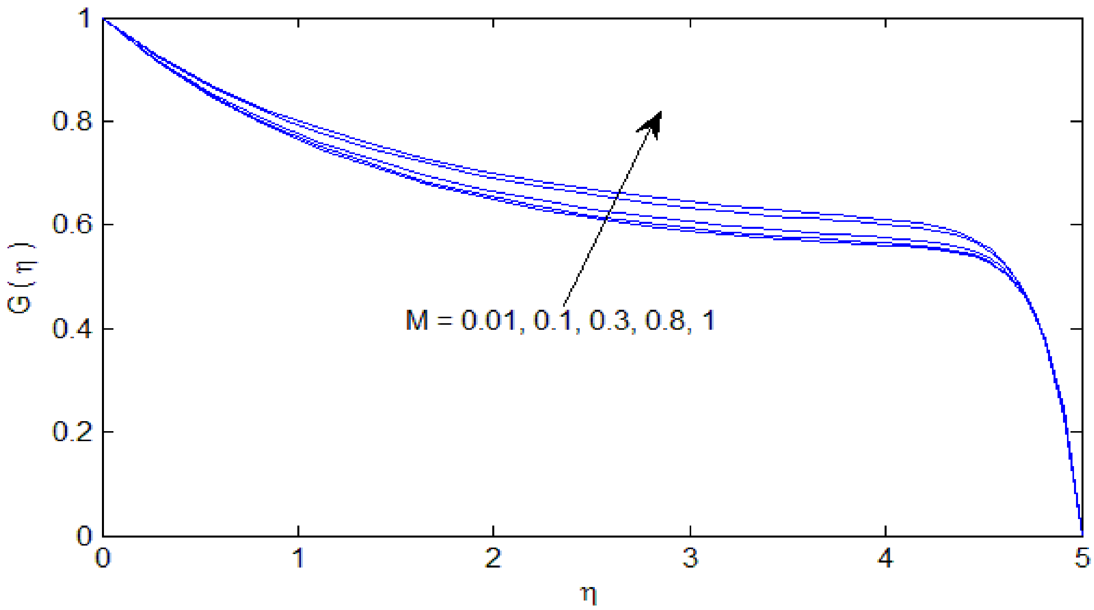


**Figure 3: Thermal boundary layer domain under the effect of magnetic field**

## Conclusion

Similarity solution is produced for a free convective boundary layer flow of electrically conducting Non-Newtonian fluids over a vertical porous and elastic surface. The governing system of Partial differential equations transformed into the system of Ordinary differential equations subject to the similarity requirement, by employing the derived transformations. Numerical solutions for special Non-Newtonian fluid so-called Powell-Eyring fluid are produced by MATLAB computational algorithm. Effect of magnetic field on the flow activity is interesting. It is observed that both velocity and temperature of fluid decrease as magnetic field increase where as slope of velocity profile and skin friction is decrease for all values of M. All the numerical solutions are generated for dimensionless quantity and hence it is executed for all types of under considered fluids.

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