Peer Reviewed Bi-Monthly Free Journal ISSN : 2320-2971 (Online)

International e-Journal for Education and Mathematics

www.iejem.org vol. 09, No. 06, (Dec. 2021), pp 1-10

Similarity Solution for MHD Non-Newtonian Fluids of Laminar Natural Convective Boundary Layer Flow

***Pankaj Sonawane*[*]*,M. G. Timol*[**]**

**Department of Mathematics, Dhanaji Nana Mahavidyalaya, Faizpur, Dist Jalgaon, India*

***Department of mathematics, Veer Narmad South Gujarat University, Surat. India*

Article Info.

Received on 3 Sep 2021

Revised on 8 Nov 2021

Accepted on 18 Nov 2021

Keywords:

MHD, Non-Newtonian Fluid, General group transformation, Similarity solution, Prandtl –Eyring fluid.

A B S T R A C T

General group transformation technique is applied to analyze the natural convection boundary layer flow of MHD Non-Newtonian fluids over a vertical flate surface. Similarity equation which is highly non-linear ordinary differential equation and is solved numerically for particular Non-Newtonian fluid. Results for velocity distribution, skin friction coefficient and temperature variation are discussed.

Introduction

The study of boundary layer flow of an electrically conducting fluid has many applications in manufacturing and natural process which include cooling of electronic devices by fans, cooling of nuclear reactors during emergency shutdown, cooling of an infinite metallic plate in a cooling bath, textile and paper industries, glass-fiber production, manufacture of plastic and rubber sheets, the utilization of geothermal energy, the boundary layer control in the field of aerodynamics, food processing, plasma studies and in the flow of biological fluids.

The mixed convection boundary layer flow of non-Newtonian fluid in the presence of strong magnetic field has wide range of application in nuclear engineering and industries. In astrophysical and geophysical studies, the MHD boundary layer flows of an electrically conducting fluid through porous media have also enormous applications. These studies are also used for modeling and simulation. Many researchers have studied the transient laminar natural convection flow past a vertical porous plate for the application in the branch of science and technology such as in the field of agriculture engineering and chemical engineering. In petroleum refineries, movement of oil, water and gas through porous media for purification and filtration are bright applied areas of research.

Duwairi and Damesh [1]studied the natural convection heat and mass transfer by steady laminar boundary layer flow over an isothermal vertical flat plate embedded in a porous medium. Mukhopadhyay et al. [2] investigated the forced convection flow and heat transfer in a porous medium using the Darcy-Forchheimer model.

A MHD flow in a duct has also been studied by Chang and Lundgren [3]. Attia and Kotb [4] investigated the two dimensional MHD flow between two porous, parallel, infinite, insulated, horizontal plates and the heat transfer through it when the lower plate was kept stationary and the upper plate was moving with uniform velocity.

Timol and Kalthia [5] is probably first to develop systematic analysis of natural convection flows of all non-Newtonian visco-inelastic fluids characterized by the special functional relationship of stress strain components.Ghosh and Shit [6] presented an interesting result on mixed convection MHD flow of Viscoelastic fluid in a porous medium past a hot vertical plate.

Similarity methods deal with reducing systems of partial differential equations into systems of ordinary differential equations. Group methods, a class of methods which lead to a reduction of the number of independent variables, were first introduced by Birkhoff [7]in 1948. Moran and Gaggioli[8,9]presented a theory which has led to improvements over earlier similarity methods. Similarity analysis has been applied intensively by Gabbert[10]. For more additional discussions on group transformation refer Ames[11, 12], Bluman and Cole[13], Boisvert et al.[14] and Gaggioli and Moran[15,16]. Throughout the history of similarity analysis, a variety of problems in science and engineering have been solved; many physical applications are illustrated by Abd-el-Malek et al.[17,18].

Recently this method has been successfully applied to various non-linear problems by Hiral and Timol[19] and Darji and Timol[20].Jain, Darji and Timol [21] presented an interesting result on natural convection boundary layer flow of non-Newtonian Sutterby fluids. So motivated by this, we produce similarity solution via group transformation technique for MHD Non-Newtonian fluids of laminar natural convective boundary Layer flow.

Governing equations

The basic equations of continuity, momentum and heat transfer of two dimensional steady laminar natural convection flow over a vertical permeablesurface in the presence of transverse magnetic field with a Cartesian coordinate system in usual notations as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\tau_{yx}\right)+g\beta\theta-\mathrm{MB}^2\,u \tag{2}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\alpha\frac{\partial^2 \theta}{\partial y^2} \tag{3}$$

With the stress-strain relationship

$$\zeta\left(\tau_{yx},\frac{\partial u}{\partial y}\right)=0 \tag{4}$$

With boundary conditions

$$y=0, \qquad u=v=0, \quad \theta=\theta_w$$

$$(5)$$

$$y=\infty, \qquad u=0 \quad \theta=0$$

Where α is thermal diffusivity, β is the volumetric thermal expansion coefficients,$M=\frac{\sigma}{\rho}U_\infty$ is magnetic parameter.

Formulation of the problem

Introducing the dimensionless quantities as

$$x^*=\frac{G_r}{L}x\ ,y^*=\frac{y}{L}(\frac{R_e}{3}G_r)^{\frac{1}{2}}\ ,u^*=\frac{u}{U_\infty}\ ,R_e=\frac{U_\infty L}{v}$$

$$v^*=\frac{v}{U_\infty}(\frac{R_e}{3G_r})^{\frac{1}{2}}\ ,\tau_{yx}^*=\frac{\tau_{yx}}{\rho U_\infty^2}(\frac{R_e}{3G_r})^{\frac{1}{2}}\ ,\theta^*=\frac{\theta}{(\theta_w-\theta_\infty)}$$

$$\theta_w^*=\frac{\theta_w}{(\theta_w-\theta_\infty)}\ ,P_r=\frac{U_0 L}{\alpha R_e}\ ,G_r=\frac{L}{U_\infty^2}g\beta(\theta_w-\theta_\infty) \tag{6}$$

Substitute the values in equation (1) to (5) and the asterisks (for simplicity),

We get

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \tag{7}$$

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=\frac{1}{\rho}\frac{\partial}{\partial y^*}\left(\tau^*{}_{y^*x^*}\right)+\theta^*-M\mathrm{B}^2u^* \tag{8}$$

$$u^*\frac{\partial \theta^*}{\partial x^*}+v^*\frac{\partial \theta^*}{\partial y^*}=\frac{1}{3P_r}\frac{\partial^2\theta^*}{\partial {y^*}^2} \tag{9}$$

With the stress-strain relationship

$$\zeta\left(\tau^*{}_{yx},\frac{\partial u^*}{\partial y^*}\right)=0 \tag{10}$$

With boundary conditions

$$y=0,\qquad u^*=v^*=0,\qquad \theta^*=\theta_w \tag{11}$$

$$y=\infty,\qquad u^*=\theta^*=0 \tag{12}$$

Introducing stream function ψ such that,

$$u^*=\frac{\partial\psi}{\partial y}\qquad ,v^*=-\frac{\partial\psi}{\partial x} \tag{13}$$

Equation of continuity (1) gets satisfied identically, equation (8-12) become

$$\frac{\partial\psi^*}{\partial y^*}\frac{\partial^2\psi^*}{\partial x^*\partial y^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial^2\psi^*}{\partial {y^*}^2}=\frac{\partial}{\partial y^*}\left(\tau^*_{yx}\right)+\theta^*-M\mathrm{B}^2\frac{\partial\psi^*}{\partial y^*} \tag{14}$$

$$\frac{\partial\psi^*}{\partial y^*}\frac{\partial\theta^*}{\partial x^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial\theta^*}{\partial y^*}=\frac{1}{3P_r}\frac{\partial^2\theta^*}{\partial {y^*}^2} \tag{15}$$

$$\zeta\left(\tau^*{}_{yx},\frac{\partial^2\psi}{\partial y^2}\right)=0 \tag{16}$$

With boundary conditions,

$$y=0,\qquad \frac{\partial\psi}{\partial y}=\frac{\partial\psi}{\partial x}=0,\qquad \theta^*=\theta_w \tag{17}$$

$$y=\infty,\qquad \frac{\partial\psi}{\partial y}=0\,,\qquad \theta^*=0 \tag{18}$$

Solution of the problem

The problem will be solved by applying linear group transformation technique to the partial differential equations (14)-(16).By the application of this transformation, the two independent variables(x,y) are reduced by one and the differential equation (14)-(16) will transforms into the ordinary differential equations.

For the present problem we introduce one parameter group of transformation given below:

$$\bar{x}^* = \Gamma^{\alpha_1} x^* \ , \qquad \bar{y}^* = \Gamma^{\alpha_2} y^*, \qquad \bar{\tau}^*_{yx} = \Gamma^{\alpha_3} \tau^*_{yx}$$

$$\bar{\psi}^* = \Gamma^{\alpha_4} \psi^* \ , \qquad \bar{\theta}^* = \Gamma^{\alpha_5} \theta^* \bar{B}^* = \Gamma^{\alpha_6} B^* \tag{19}$$

Where$\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6and\Gamma$ are constants.

Introducing the linear transformation, given by equation (19), into the Equations (14)-(16) and then we obtained similarity variable as follow,

$$\eta = \frac{y^*}{x^{*\frac{1}{3}}} \ , \qquad \psi^* = f(\eta) x^{*\frac{2}{3}}, \qquad \theta^* = G(\eta) x^{*-\frac{1}{3}} \tau^*_{yx} = H(\eta) \ \ and \ B = B_0 x^{-\frac{1}{3}} \tag{20}$$

The reduction to an ordinary differential equation

The similarity variables (20) maps equations (14)-(18) in to the following non-linear ordinary differential equations

$$f'^2(\eta) + 2f(\eta)f''(\eta) - 3H'(\eta) - 3G(\eta) + 3Mf'(\eta) = 0 \tag{21}$$

$$2f(\eta)G'(\eta) + f'(\eta)G(\eta) + \frac{1}{Pr} G''(\eta) = 0 \tag{22}$$

With stress-strain relationship is given by,

$$\zeta(H, f'') = 0 \tag{23}$$

With the boundary condition

$$\eta = 0 \ , \qquad f'(0) = 0 \ , f(0) = 0 \ \ , G(0) = 1 \ \ subject to \theta_w = 1$$

$$\eta \to \infty \ , \qquad f'(\infty) = 0, \qquad G(\infty) = 0 \tag{24}$$

Results and Discussion

For finding the numerical solution we have consider the Prandtl-Eyring fluid model. Mathematically this model is givenas,

$$\tau_{yx} = A \sin h^{-1} \left(\frac{1}{B} \frac{\partial u}{\partial y} \right) \tag{25}$$

Where*A* and *C* are flow consistency indices introducing the dimensionless quantities.

Introducing the dimensionless quantities

$$H'(\eta) = \alpha f'''(1+\gamma(f'')^2)^{-\frac{1}{2}} \tag{26}$$

Where $\alpha = \frac{A}{\mu_\infty C}$; $\gamma = \frac{\rho u_\infty^3}{\mu L C^2}$ are dimensional flow parameters.

Substituting it into the equation (21), we get

$$f''' = \frac{1}{3\alpha}\left(f'^2 + 2ff'' - 3G + 3MF'\right)\left(1+\lambda F''^2\right)^{\frac{1}{2}} \tag{27}$$

and the physical quantity of interest are the coefficient of local skin friction

$$C_f = \frac{2\tau_{yx}}{\sqrt{R_e}} \tag{28}$$

Where $\tau_{yx} = \frac{\alpha}{\sqrt{\gamma}} \sin h^{-1}\left(\beta^{\frac{1}{2}} F''(0)\right)$

The numerical method applied to solve equation (27) and (22) with the boundary conditions (24)are obtained using MATLAB ode solver.Figures 1-3 shows thatMagnetic fields effect M on velocity profile, local shear stress and temperature profile. When increase in magnetic field $M = 0.1, 0.3, 0.5, 0.8$ causes the boundary layers to thicken. Figures 4-6 shows the effect of flow parameters α and γ on similarity variables related to velocity, local skin friction and temperature. The boundary layer thickness decreases as the flow parameter α increases by controlling flow parameter γ and M. Figures 7-9 shows that the influence of Prandtl number on fluid flow. These figures show an increase in Prandtl number causing the boundary and thermal boundary layer decreases thickness

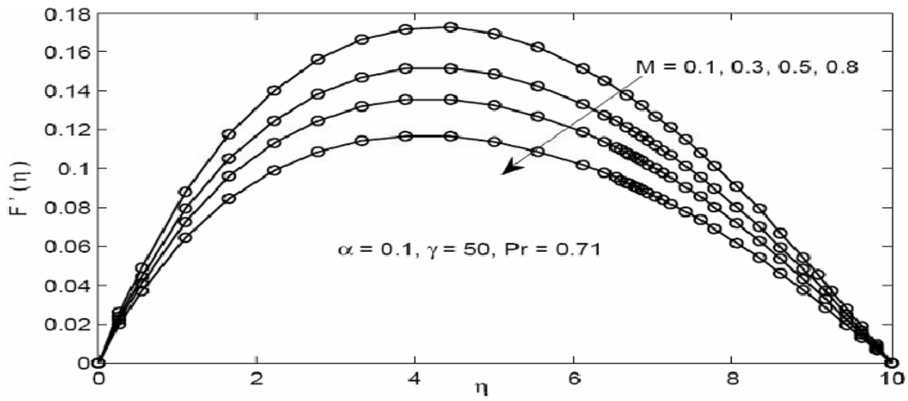

Figure1.Magnetic field effects on local velocity profile

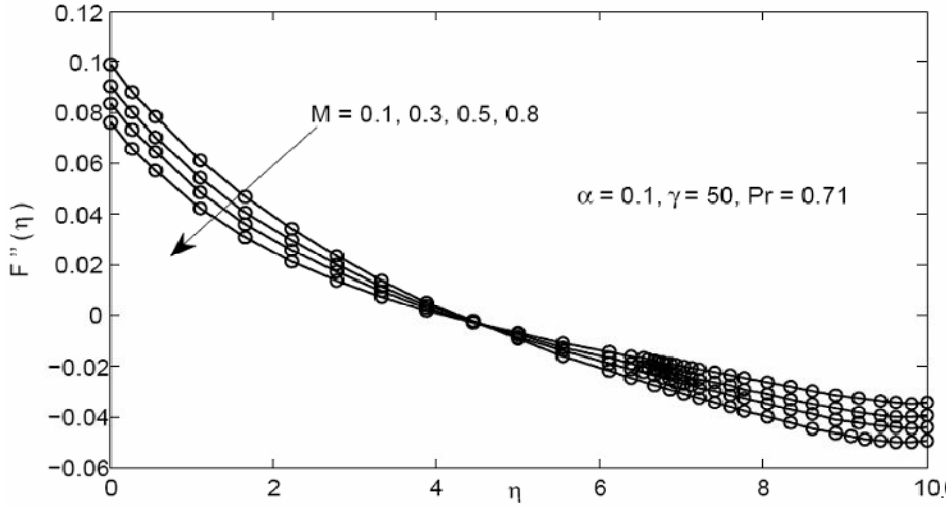

Figure 2. Magnetic field effects on local shear-stress profile

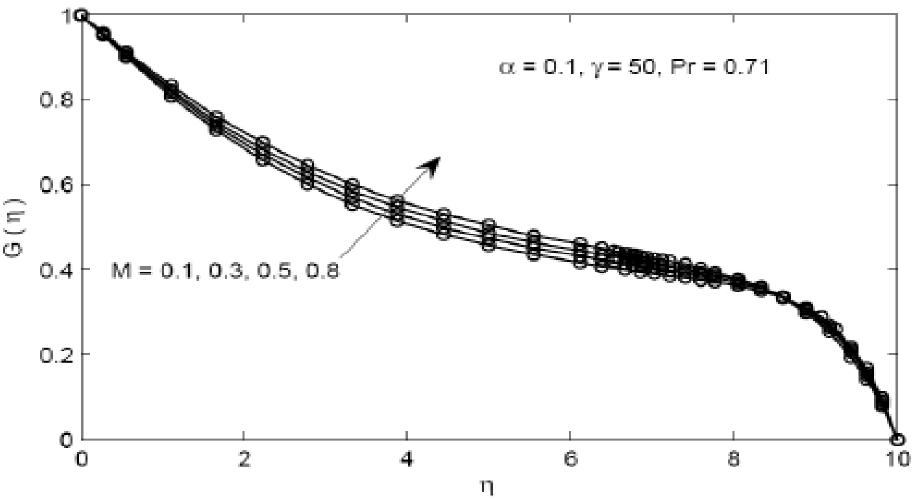

Figure 3. Magnetic field effects on temperature profile

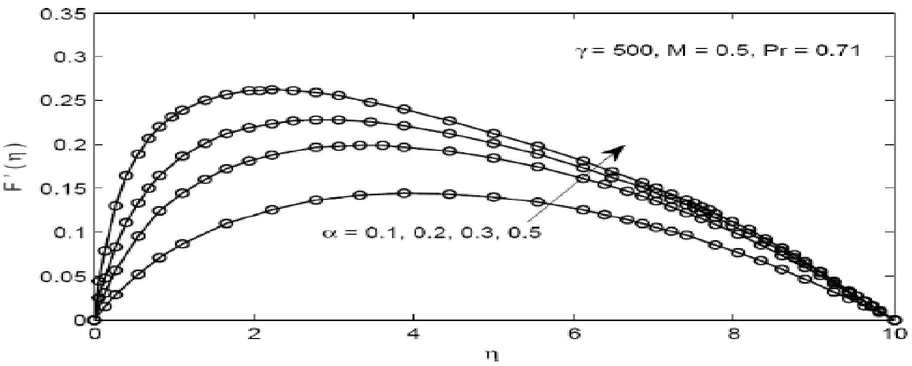

Figure 4(a)

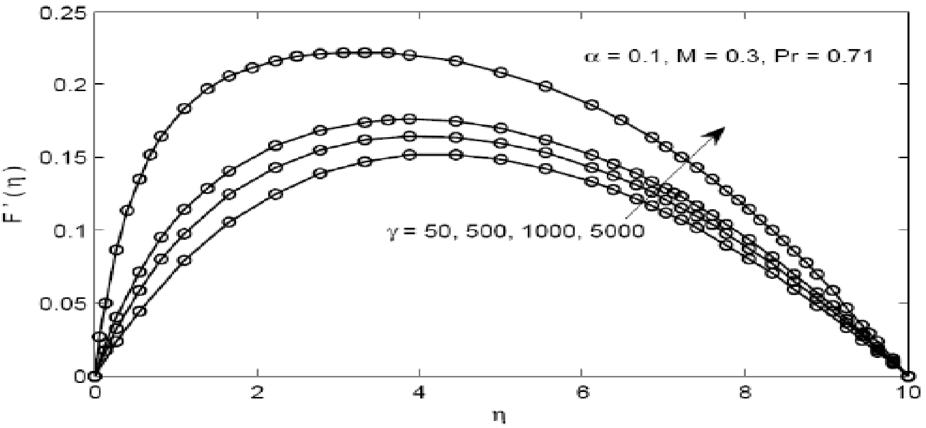

Figure 4(b)

Figure 4.Influence of flow parameter on non-dimensional velocity

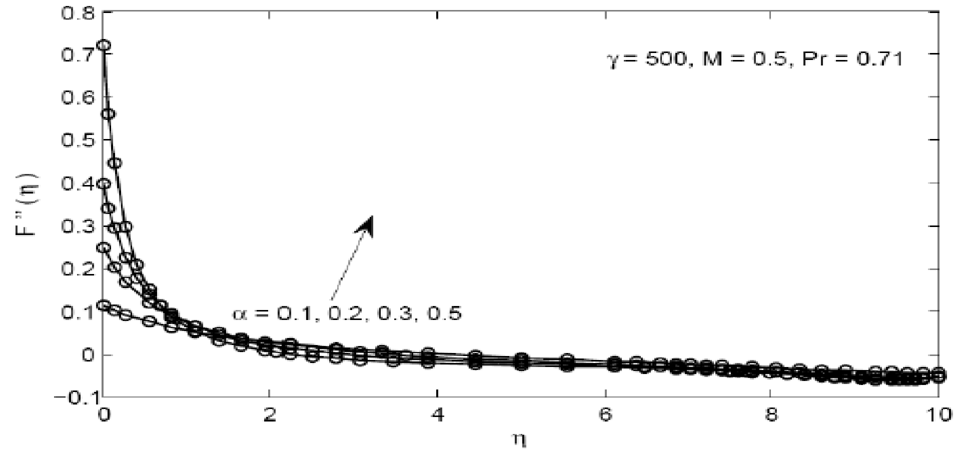

Figure 5(a)

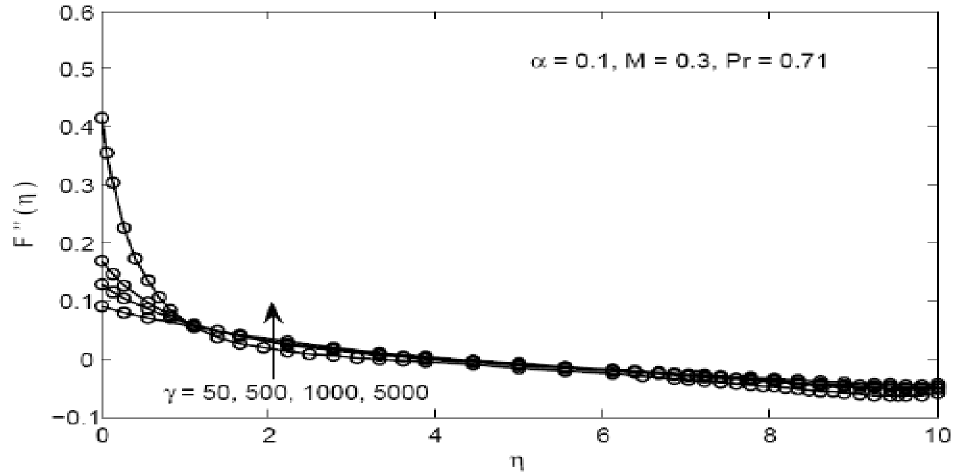

Figure 5(b)

Figure 5 Influence of flow parameter on non-dimensionallocal shear-stress

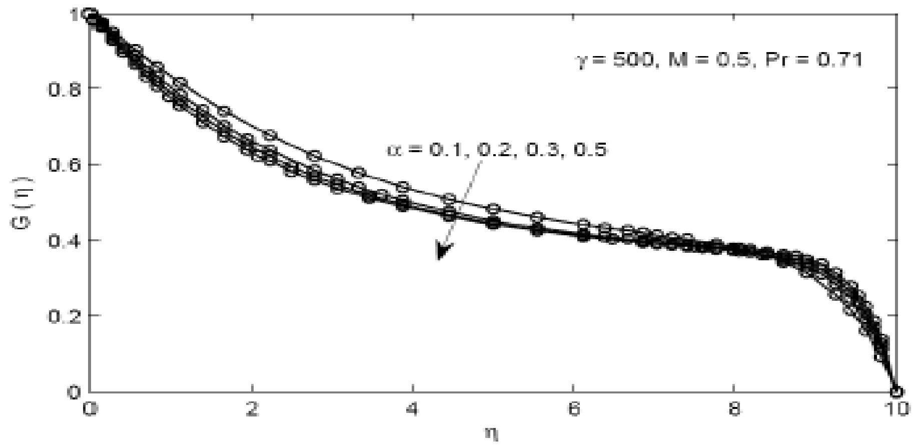

Figure 6(a)

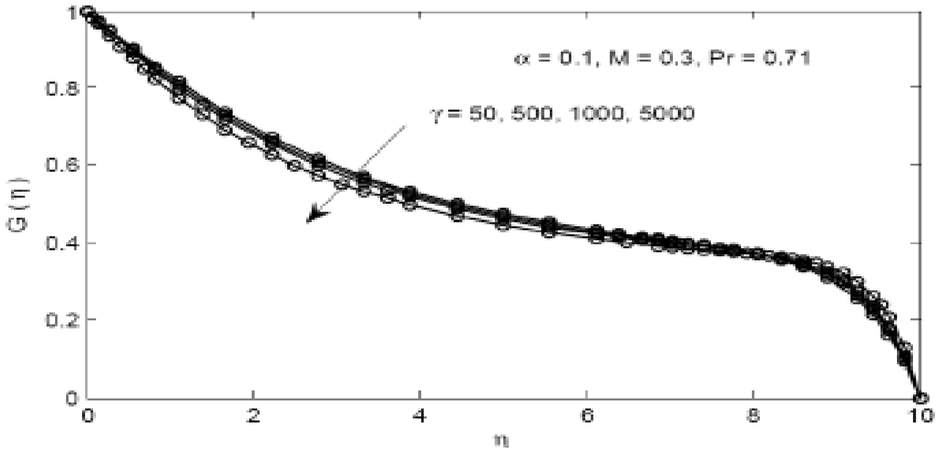

Figure 6(b)

Figure 6 shows that influence of flow parameter on non-dimensional temperature

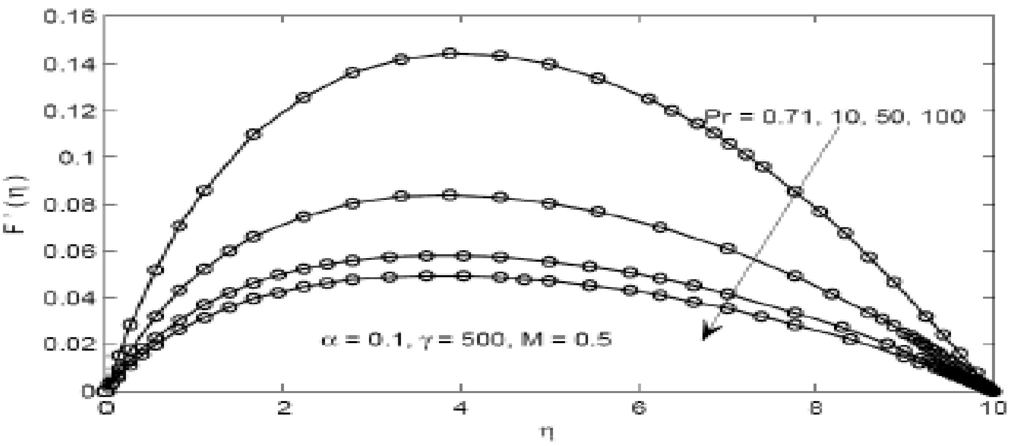

Figure 7 Effect of Prandtl number on dimensionlessvelocity of Prandtl Eyring fluid

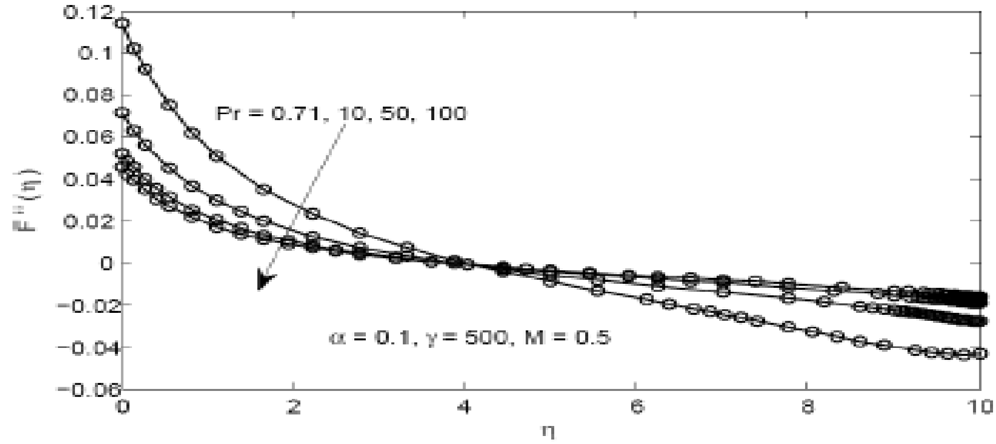

Figure 8 Effect of Prandtl number on skin-friction coefficient of Prandtl Eyring fluid

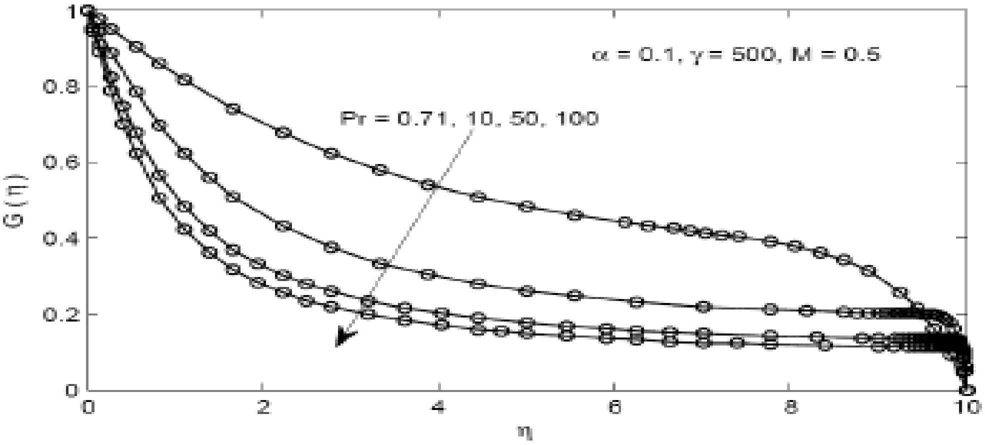

Figure 9 Temperature variations under the effect of Prandtl number on Prandtl Eyring fluid

Conclusion

Linear group transformation technique is applied to analyze the natural convection boundary layer flow of MHD Non-Newtonian fluids. The governing partial differential equations are converted into ordinary differential equations by using group transformation technique. Numerical solutions are presented in a graphical form for Prandtl-Eyring fluids. Effect of magnetic field, influence of flow parameter and Prandtl number are discussed briefly. The present results are same as those available in Panigrahi [22], Na and Hansen [23] and Hayet et al. [24].

References:

1. Duwairi HM, Damseh RA. Thermophoresis particle deposition-thermal radiation interaction on mixed convection from vertical surfaces embedded in porous medium. Can J Phys; 87: 161,2009.

2. Mukhopadhyay S, Layek GC. Radiation effect on forced convective flow and heat transfer over a porous plate in a porous medium. Meccanica;44: 587-597,2009.

3. Chang C C and Lundgren T S; Duct Flow in Magneto Hydrodynamics Zeitschrift für Angewandte Mathematics und Physik, 12, 100-114,1961.

4. Attia H A and Kotb N A; MHD Flow between Two Parallel Plates with Heat Transfer Acta Mechanica, 117, 215-220,1996.

5. Timol M.G. and Kalthia N. L., Group theoretic approach to similarity solutions in non-Newtonian natural convection flows, Reg. J. Energy Heat and Mass Trans., Vol.7, No.4, pp.251-288.
6. Ghosh S.K., Shit G.C., Mixed convection MHD flow of Viscoelastic fluid in a porous medium past a hot vertical plate, World Journal of Mech,2, pp262-271,2012

7. G. Birkhoff, Mathematics for engineers, Elect. Eng.67 ,1185,1948.

8. R.A. Gaggioli, M.J. Moran, Group theoretic techniques for the similarity solution of systems of partial differential equations with auxiliary conditions, Mathematical Research Center, University of Wisconsin, Technical Summary Report No. 693, 1966.

9. M.J. Moran, R.A. Gaggioli, A new systematic formalism for similarity analysis, J. Eng. Math. 3 ;151–162,1968.

10. C.H., Gabbert, Similarity for unsteady compressible boundary layers, AIAA J. 5 No. 6.1967.

11. W.F. Ames, Similarity for nonlinear diffusion equation, I &Ec Fund. 4 ,72–76,1965.
12. W.F. Ames, M.C. Nucci, Analysis of fluid equations by group methods, J. Eng. Math. 20 ,181–187,1985.

13. G.W. Bluman, J.D. Cole, Similarity Methods of Differential Equations, Springer, New York, 1974.

14. R.F. Boisvert,W.F. Ames, U.N. Srivastava, Group properties and new solutions of Navier–Stokes equations, J. Eng. Math. 17 ,203–221,1983.

15. R.A. Gaggioli, M.J. Moran, Similarity analysis of compressible boundary layer flows via group theory, Mathematical Research Center, Universityof Wisconsin, Technical Summary Report No. 838, 1967.

16. R.A. Gaggioli, M.J. Moran, Similarity analysis via group theory, AIAA. J. 6 ,2014–2016,1968.

17. M.B. Abd-el-Malek, M.M. Helal, Steady flow of an electrically conducting incompressible viscoelastic fluid over a heated plate, Z. Naturforsch60a , 29–36,1968.

18. M.B. Abd-el-Malek, Y.Z. Boutros, N.A. Badran, Group method analysis of unsteady free-convective boundary-layer flow on a nonisothermal vertical flat plate, J. Eng. Math. 24 (4) ,343–368,1990.

19. Hiral Parmar, M.G.Timol, Deductive Group Technique for MHD coupled heat and mass transfer natural convectionflow of non-Newtonian power law fluid over a vertical cone through porous medium, Int. J. of Appl.Math andMech. 7 (2) ,35-50,2011.

20. R. M.Darji, M. G. Timol, Invariance analysis and similarity solution of heat transfer forMHDviscous power-lawfluid over a non-linear porous surface via group theory , Int. J. of Appl.Math andMech.1 (2) ,pp116-132,2013.

21. Jain N., Darji R.M., and Timol M.G., Similarity solution of natural convection boundary layer flow of non-Newtonian Sutterby fluids, Int. J. Adv. Appl. Math. and Mech. 2 (2), pp.150-158, 2014.

22. Panigrahi, S., Reza, M. and Mishra. A. K., MHD effect of mixed convection boundary-layer flow ofPowell-Eyring fluid past nonlinear stretching surface, Applied Mathematics and Mechanics, Vol.35, No. 12, pp. 1525-1540, 2014.

23. Na, T. Y. and Hansen, A. G., Possible similarity solution of laminar natural convection flow of non-Newtonian fluids, Int. J. of heat and mass transfer, Vol 9, pp 261, 1966.

24. Hayat, T., et al., Similarity solutions for boundary layer equations of a Powel-Eyring fluid, Mathematical and Computational Applications, Vol. 18, No. 1, 62-70,2013.